Dissertation Defense

March 29th 2018

The Impact of LHC Run I pPb W/Z data on the nCTEQ15 PDF set

Eric Godat

Outline

- Introduction
  - The Standard Model
  - QCD and Phenemonology
  - Parton Distribution Functions (PDFs)
- The nCTEQ Collaboration
- PDF Reweighting
- Refitting nCTEQ15
  - nCTEQ++
  - nCTEQ+LHC
- Conclusions

Introduction

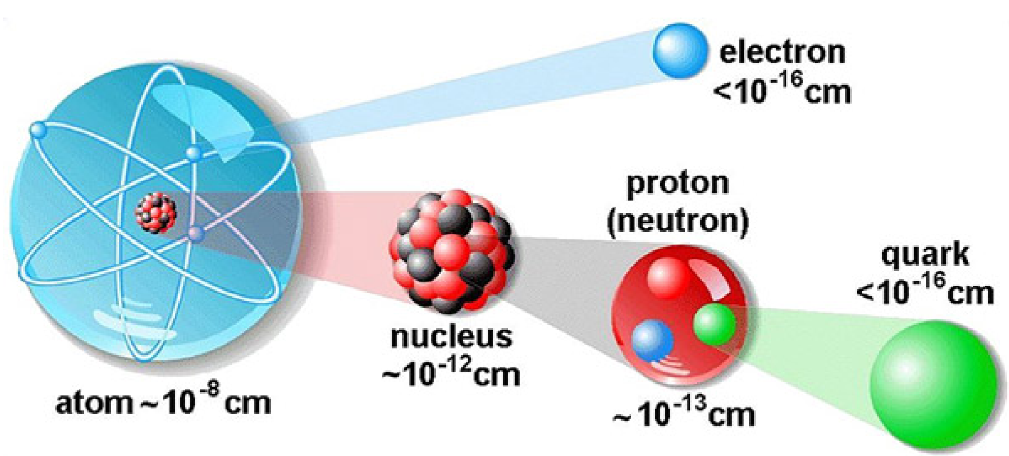
The Fundamentals

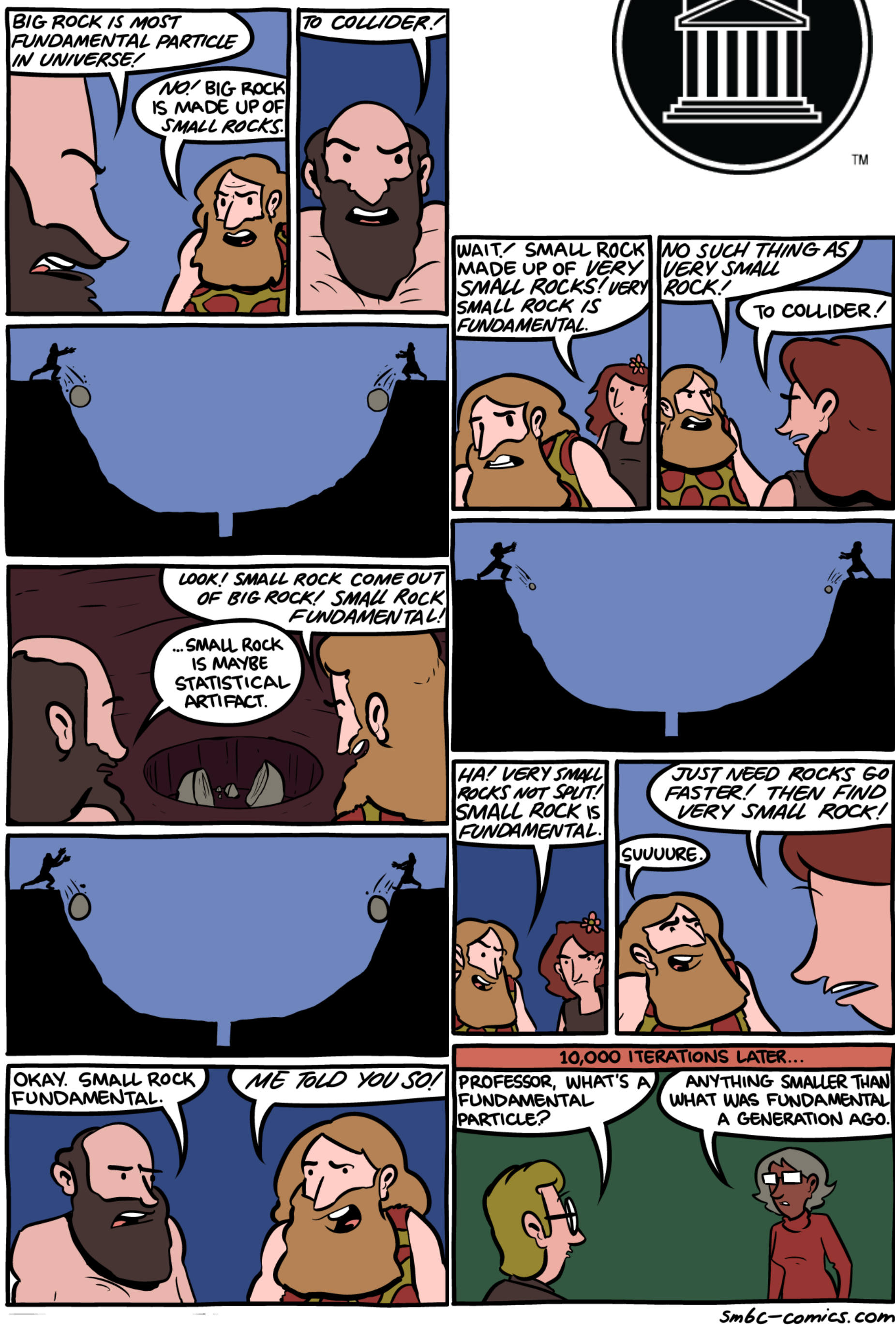
What is matter made of?

Atoms

- Electrons
- Nucleus
  - Protons/Neutrons
    - Partons (quarks)

Higher energies = Smaller distances





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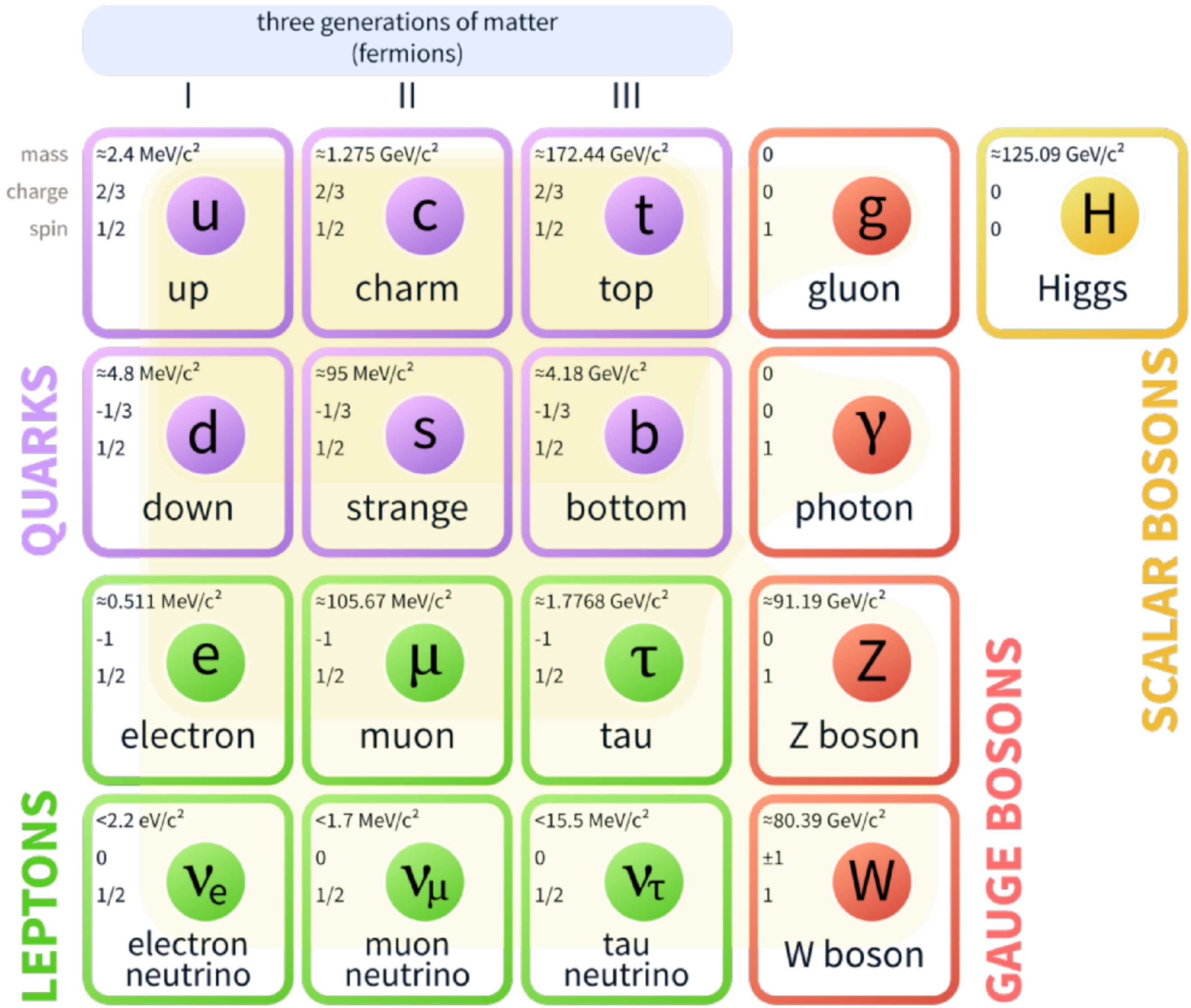
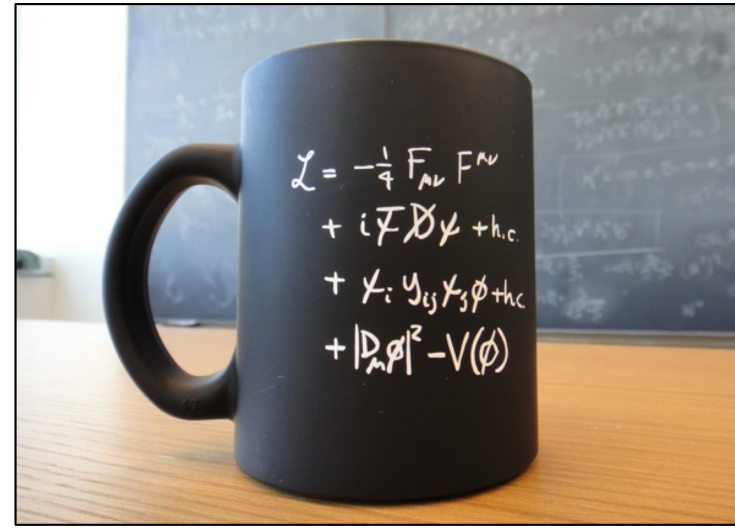
The Standard Model



$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$



Describes particles:

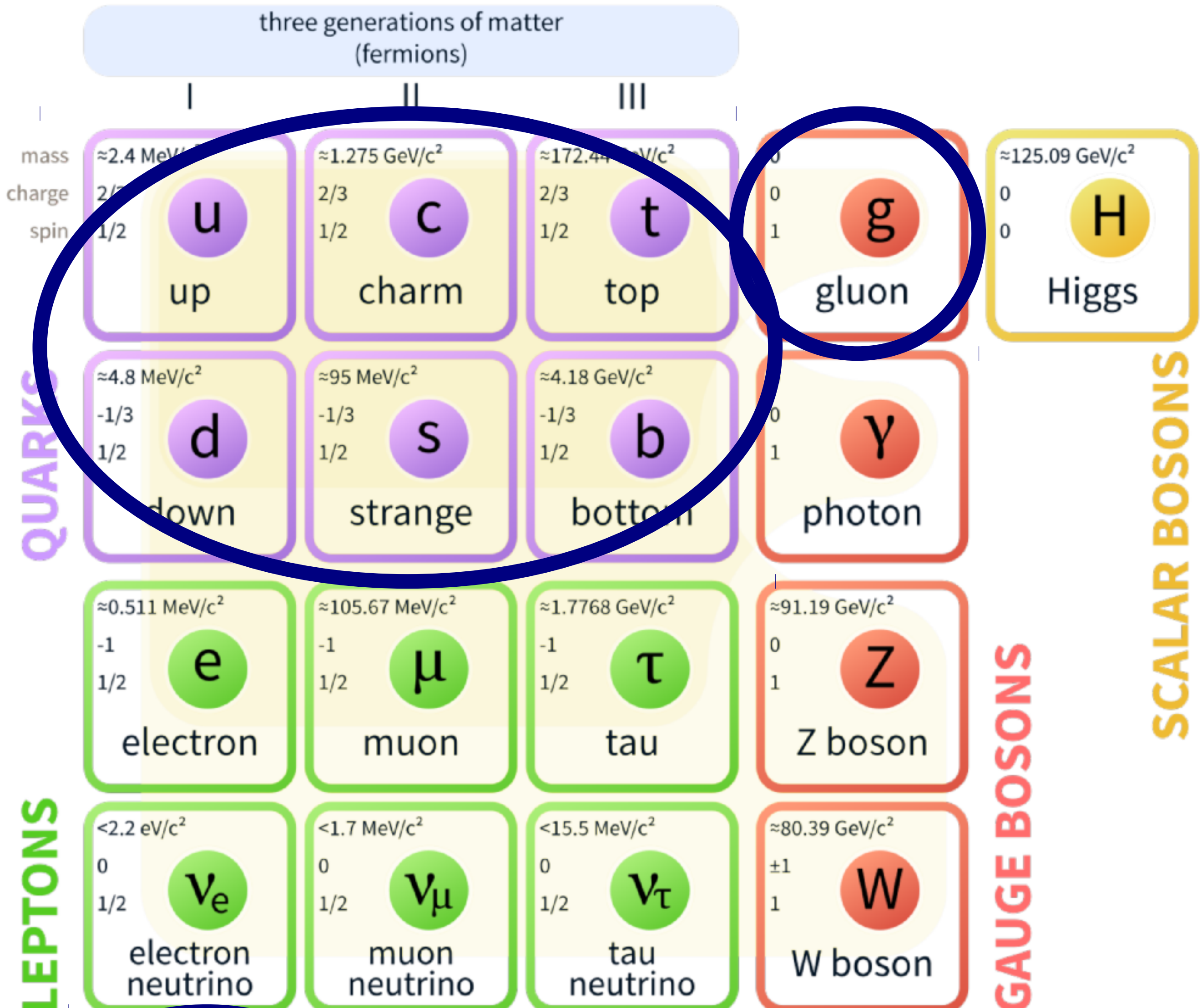
- Fermions
  - Quarks
  - Leptons
- Bosons
  - Gauge
  - Scalar

And their interactions:

- Electromagnetic Force
- Weak Force
- Strong Force
  - Neglects Gravity

Image Sources: (1)(2)

Quantum Chromodynamics (QCD)



$$SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$$

The study of proton structure is governed by QCD

A hint of color (QCD)

- Color Charge

  3 colors/ 3 anticolors

  - red, blue, green
  - anti-red, anti-blue, anti-green

  - Quarks carry a single color/anticolor
  - Gluons carry a color <u>and</u> an anti-color

- Nature is color neutral
  - Net color has not been observed
  - Confinement

$SU(3)_C$

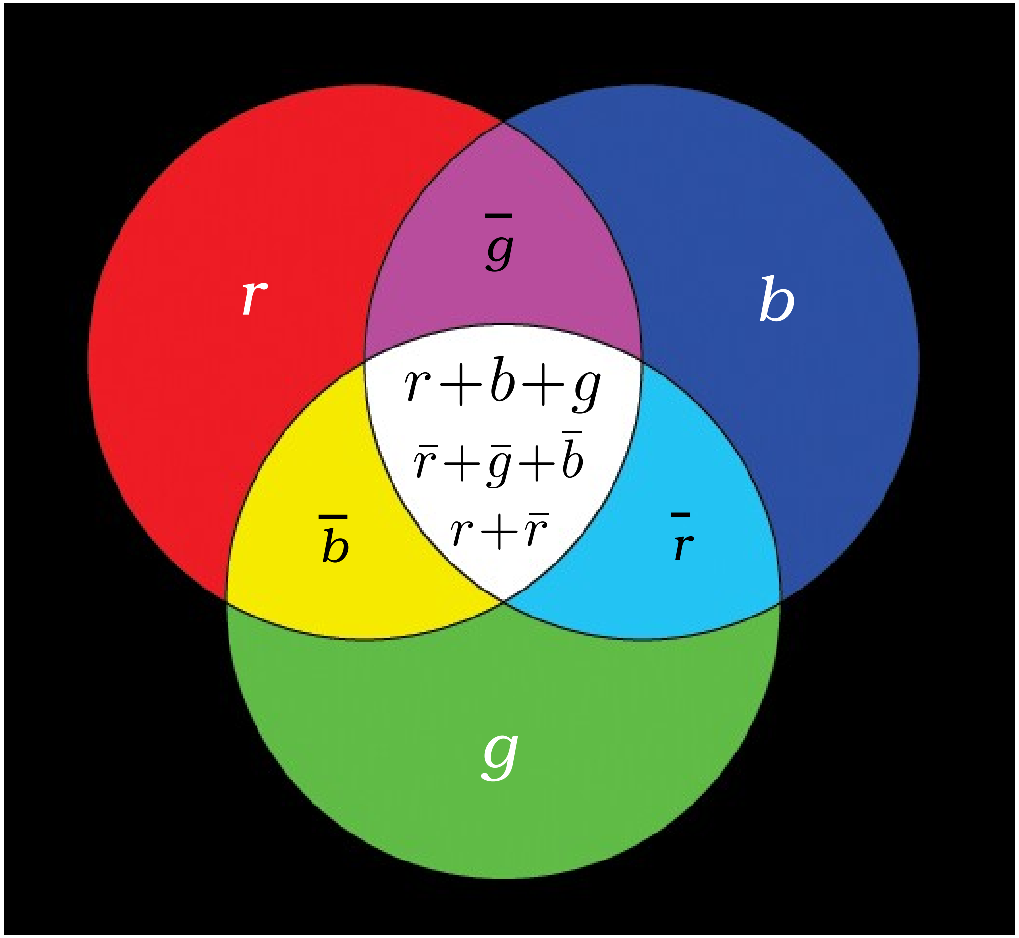
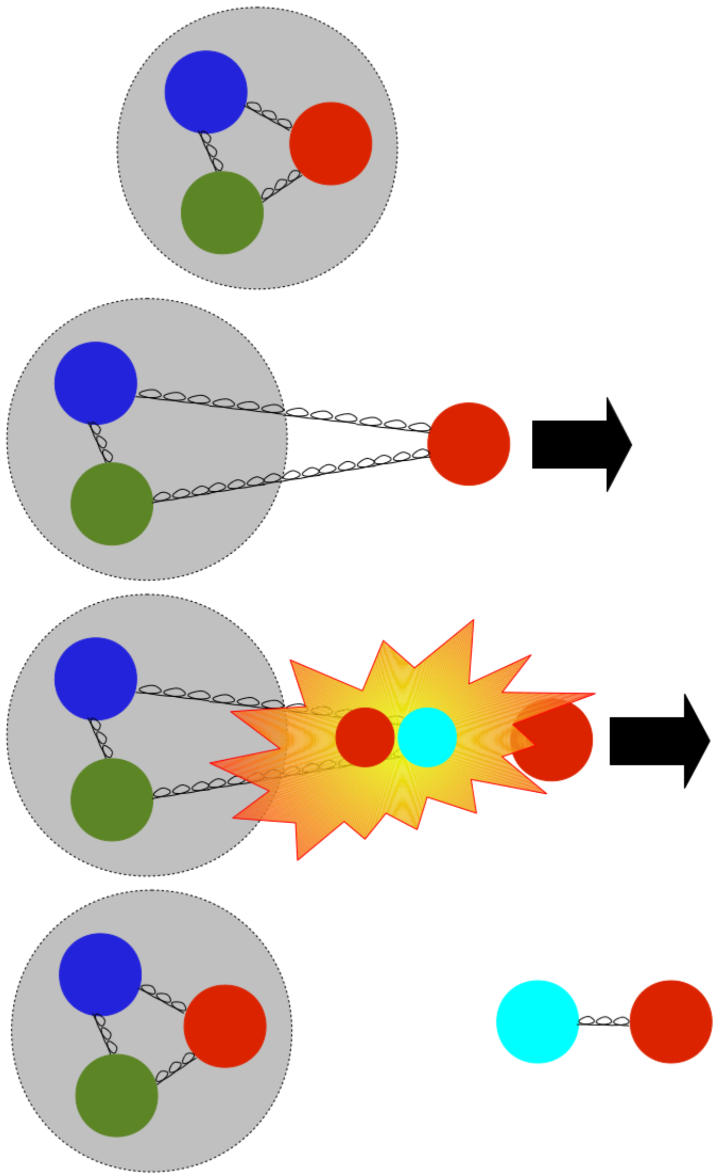
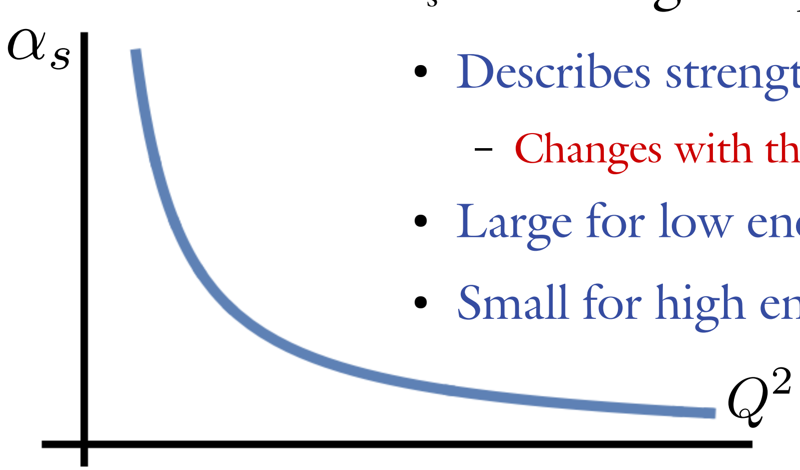


Image Source

Asymptotic Freedom (QCD)

- Why can't I make a color "ion"?
  - As long as quarks are close together, they can move freely
  - Pulling a single quark away requires exponentially more energy
  - This energy then creates a quark – anti-quark pair that satisfies confinement



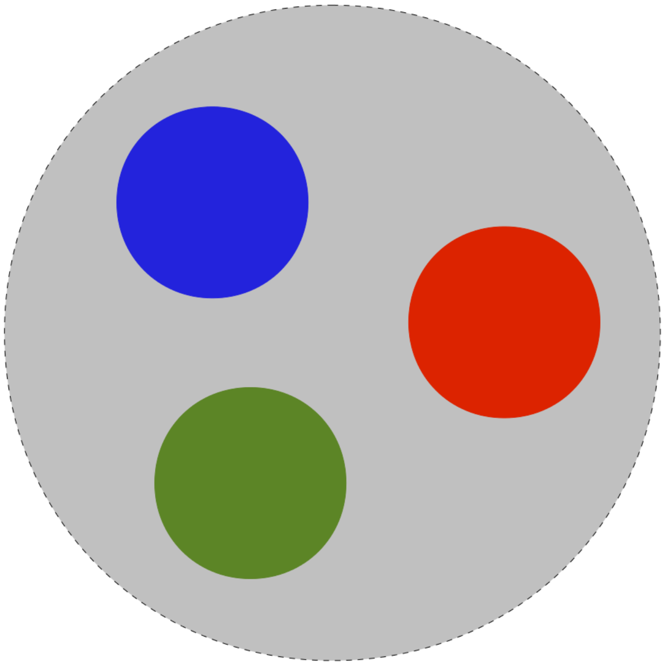


- α_s - Running Coupling Constant
  - Describes strength of QCD interactions
    - Changes with the energy of the interaction
  - Large for low energy interactions
  - Small for high energy

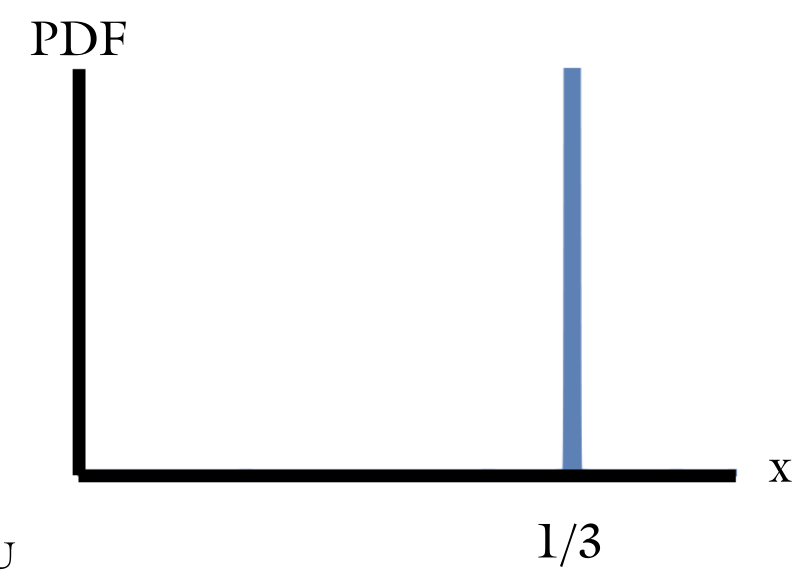
What is a Proton?

Protons are hadrons

- Made of partons
  - 3 Valence quarks (uud)
    - Determine quantum numbers



- Structure described by parton distribution functions (PDFs)



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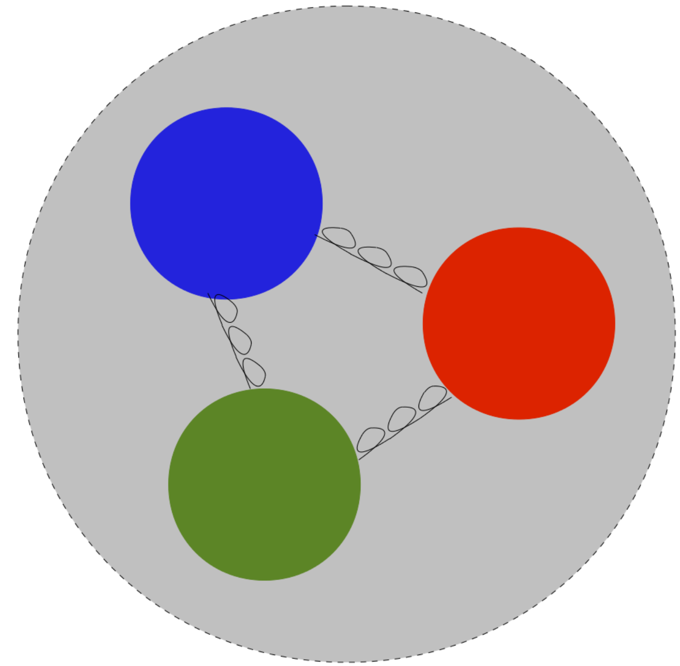

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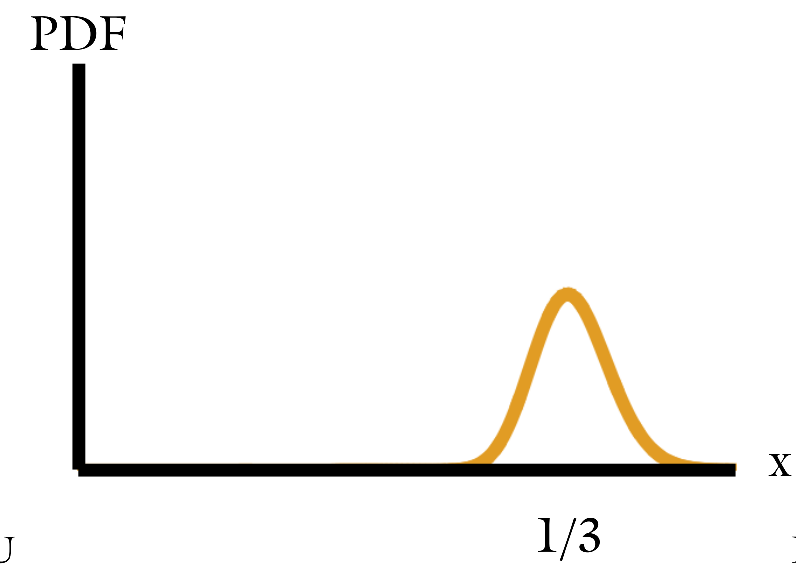

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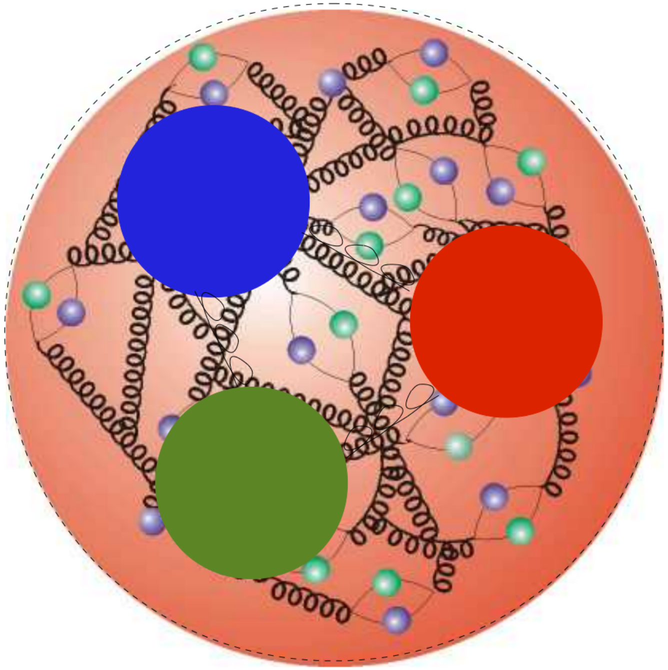
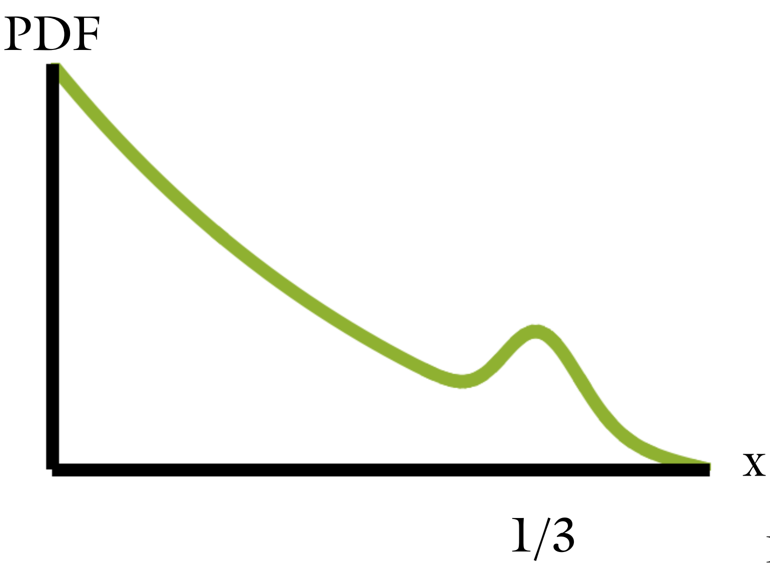
  - Sea quarks
- Structure described by parton distribution functions (PDFs)





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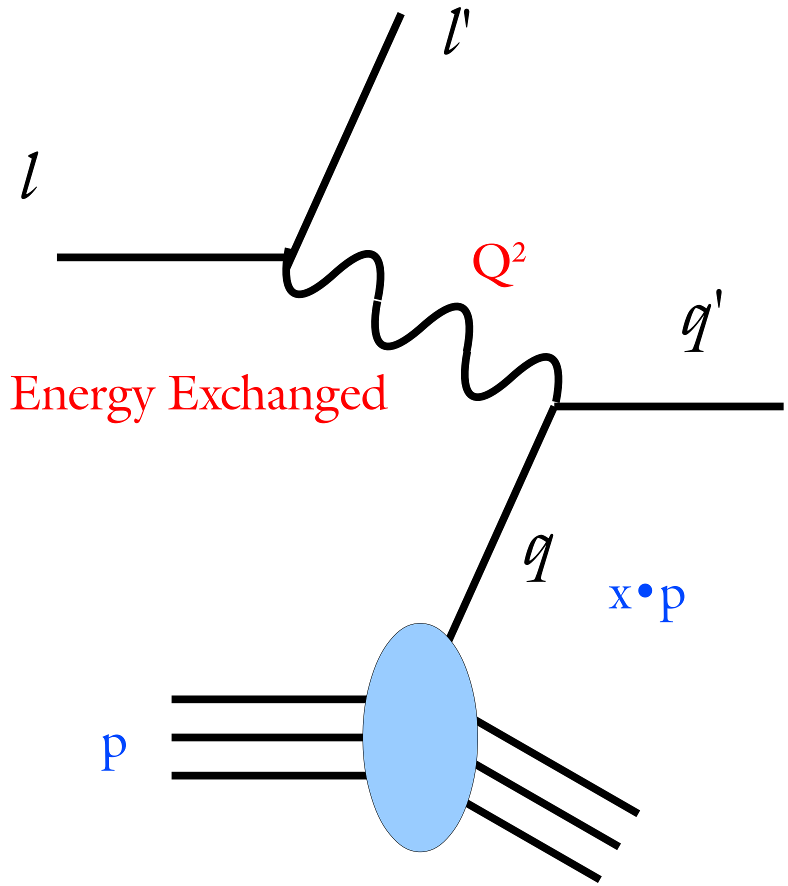
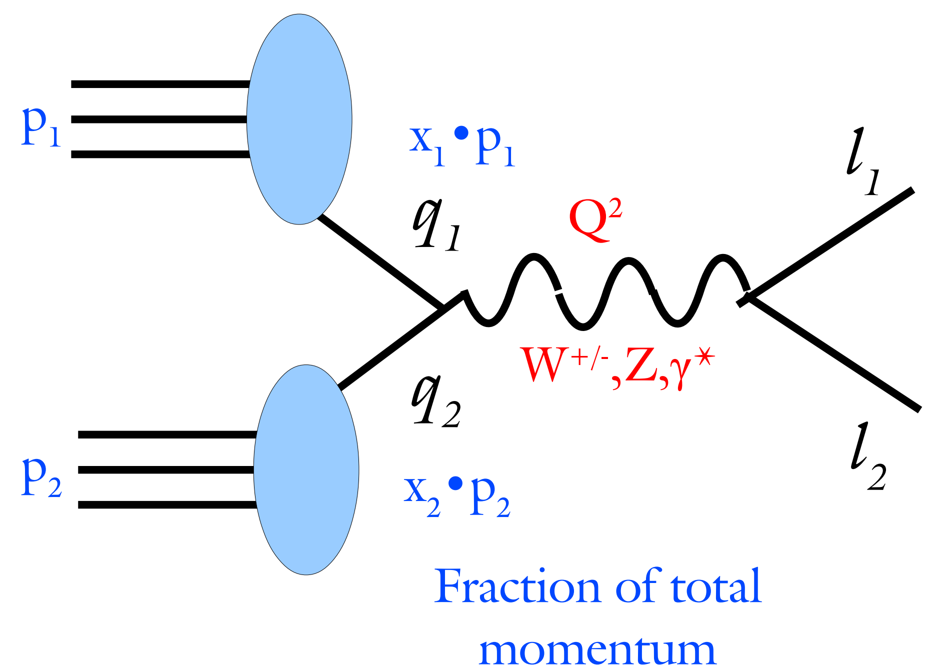
Studying Structure

Scattering experiments provide insight into the structure of the proton

Deeply Inelastic Scattering



Drell-Yan Process



Factorization

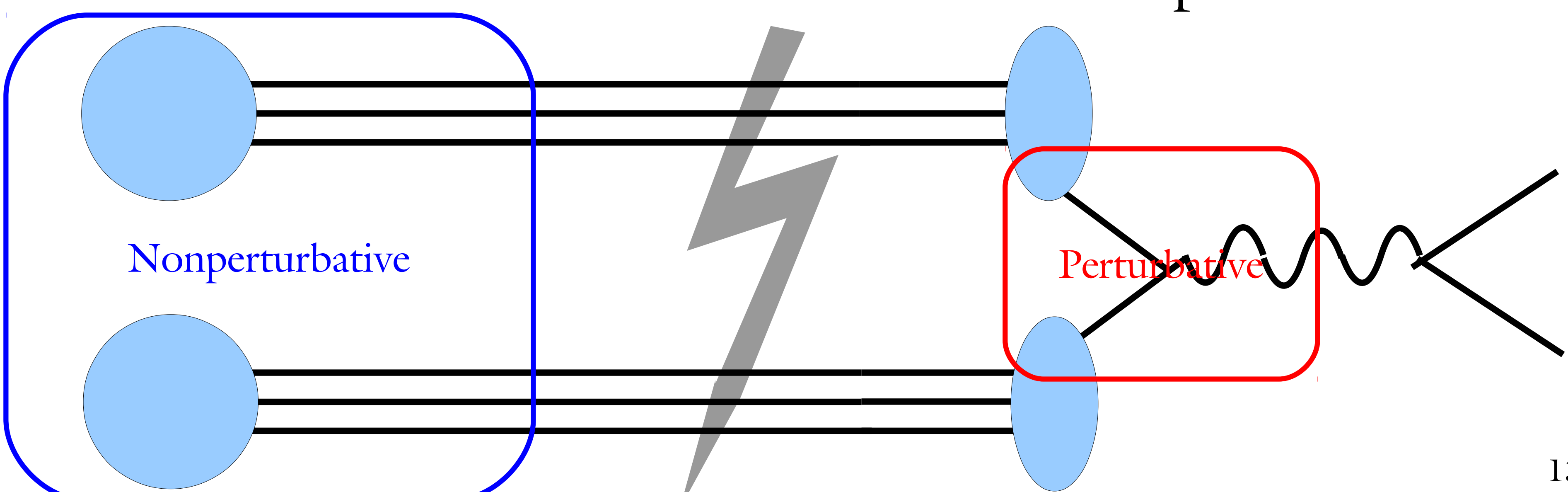
We link theory to experiment with the factorization theorem

$$\sigma_{DY} = \sum_{q_1, q_2} f_1(x, Q^2) \otimes \hat{\sigma}_{q_1 q_2} \otimes f_2(x, Q^2)$$

Nonperturbative
(Long Range)

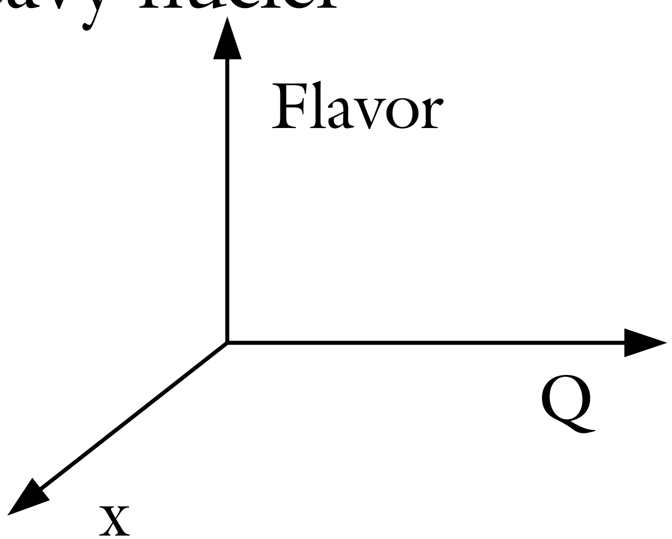
Perturbative
(Short Range)

Parton Distribution Functions describe the nonperturbative terms



Parton Distribution Functions (PDFs)

- Describe the probability of a parton inside a proton with a given momentum fraction at a given energy interacting
- Parameterized fits to experimental data
- Universal across processes
- Nuclear corrections are needed for heavy nuclei
- Broken down into discrete grids by:
  - All quark flavors + gluon
  - Hard scattering energy, Q
  - Momentum fraction, x



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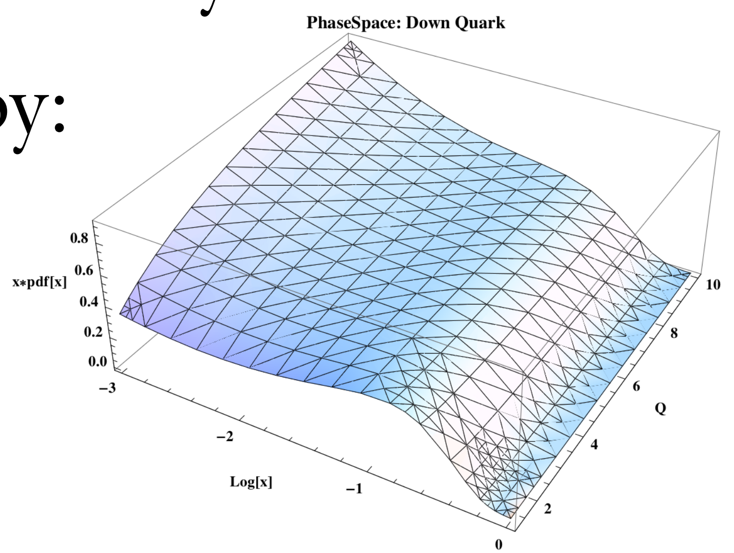


Nuclear PDFs

Nuclear corrections are needed to describe heavy nuclei

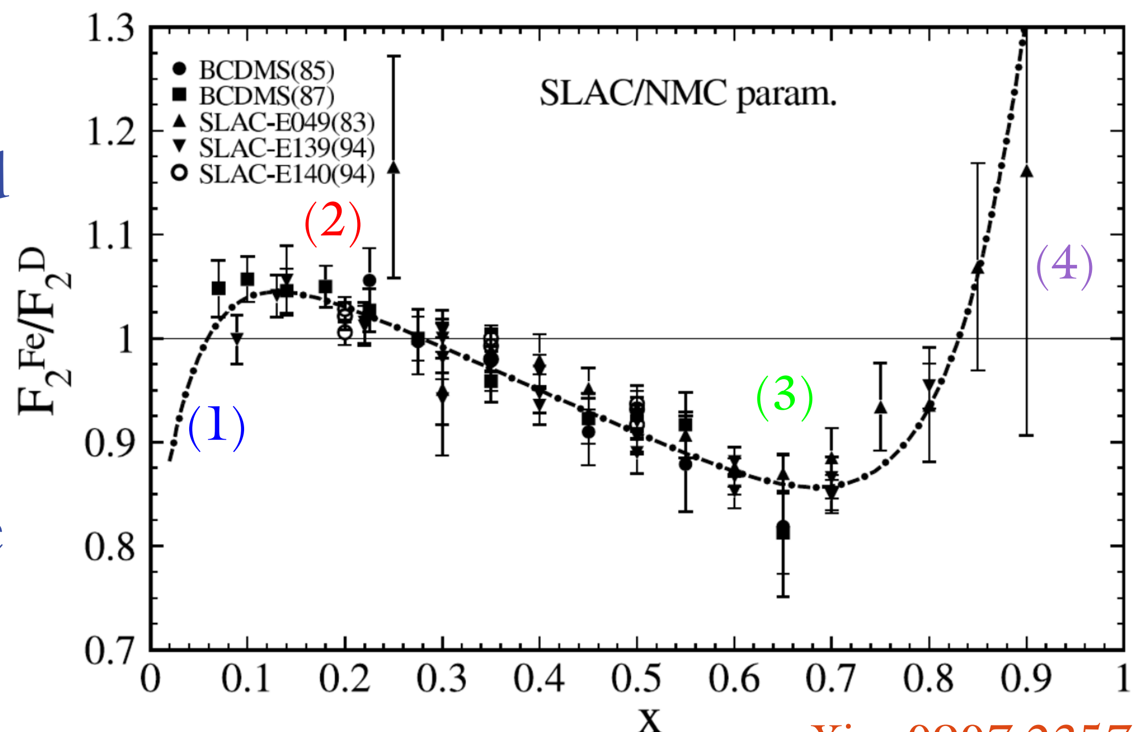
- Not just a sum of protons and neutrons
  - Partons can share momentum between nucleons
- Historically nuclear effects are described in regions of x

(1) Shadowing

(2) Anti-Shadowing

(3) EMC Effect

(4) Fermi Motion



arXiv: 0907.2357

The nCTEQ Collaboration

nCTEQ PDFs

Formalism:

- Generalized A-Parameterization
  - $xf_i^{p/A}(x, Q^2) = c_0 x^{c_1}(1-x)^{c_2} e^{c_3 x}(1+e^{c_4}x)^{c_5}$
  - $c_k \to c_{k,0} + c_{k,1}(1 - A^{-c_{k,2}})$

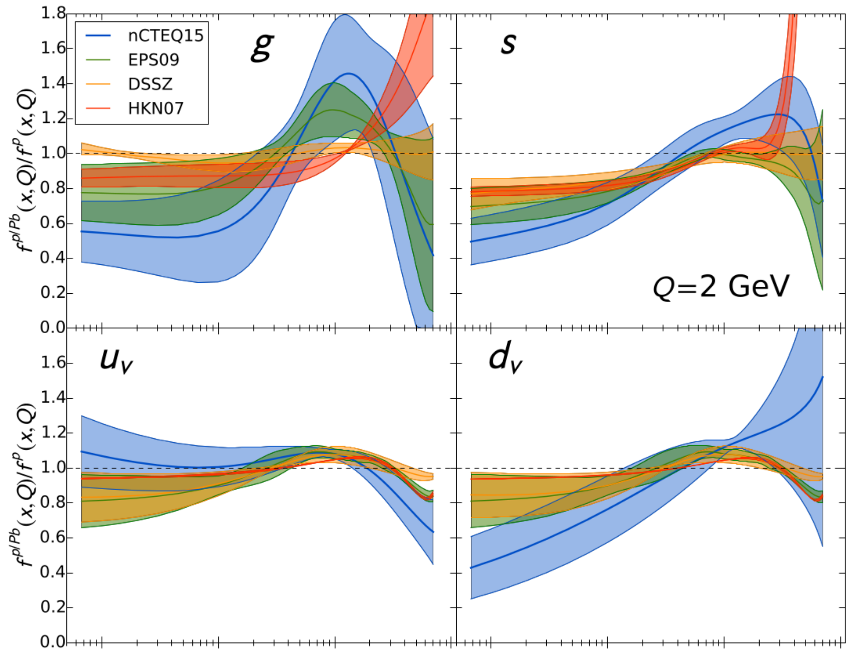
    No multiplicative nuclear correction

- More parameters than proton fits

    ~ 3 times as many so we make assumptions

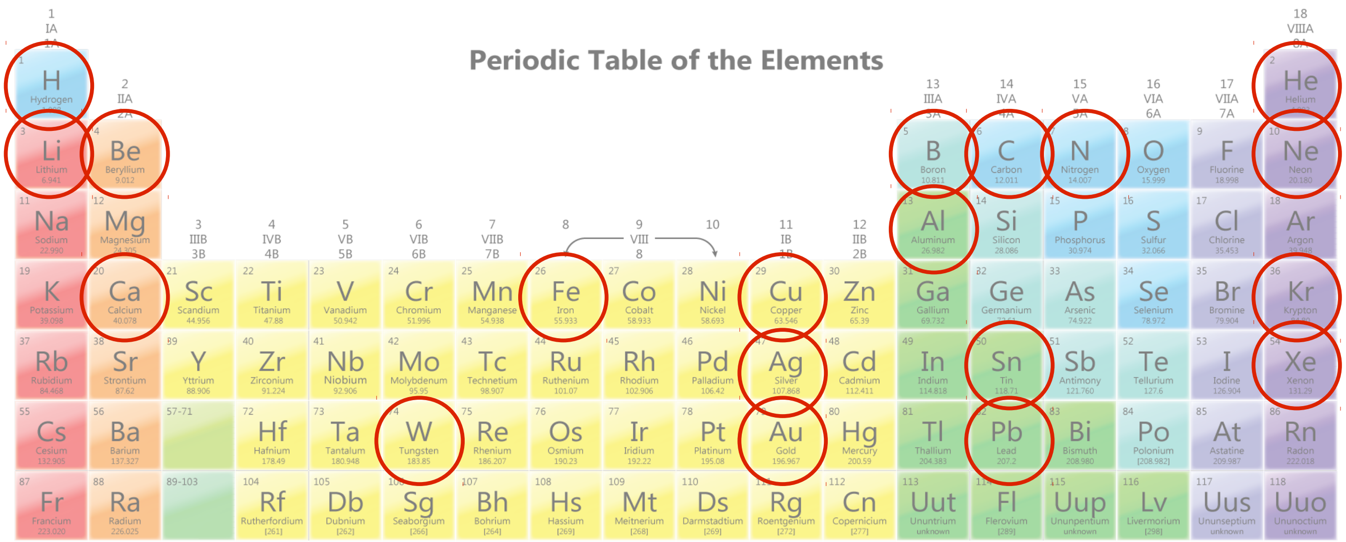
- Fewer data points

    e.g. 740 nuclear points for nCTEQ15 vs 2947 protons points for CT14[1]



[1]Ref: Dulat et Al. arXiv:1506.07443

nCTEQ PDFs



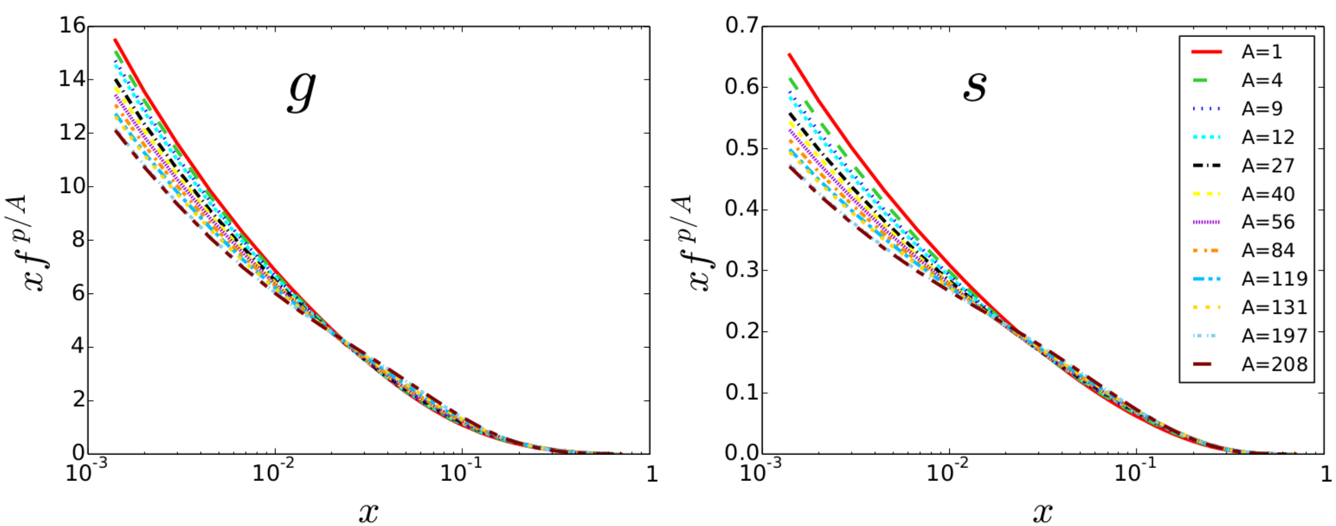
Nuclei with DIS data included in nCTEQ15

$$f^N = \frac{Z}{A} f^{p/N} + \frac{A-Z}{A} f^{n/N}$$

Assume isospin symmetry

Currently at NLO

Parameterization allows for construction of any nuclei



nCTEQ15 PDF Set

Global fit to experimental data

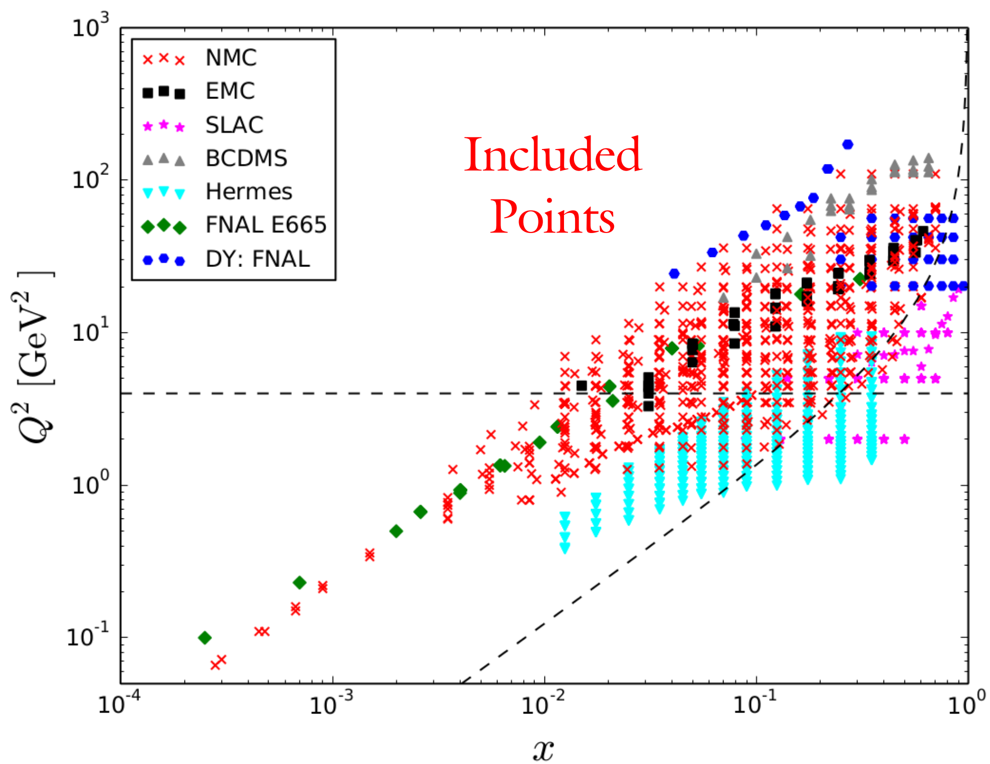
- Deep Inelastic Scattering
- Drell-Yan
- Pion Production Data

NO LHC data

740 nuclear data points after kinematic cuts

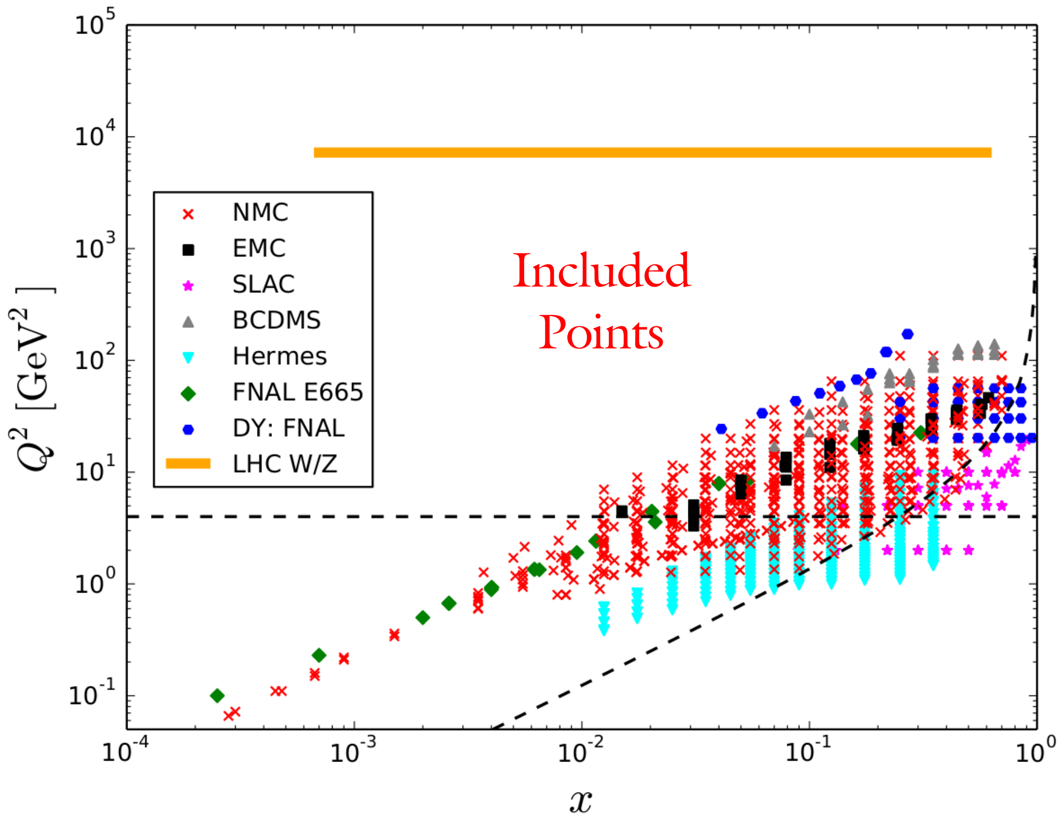
Error analysis with Hessian Method

- 16 Eigenvalues



Reweighting

LHC Data for nCTEQ



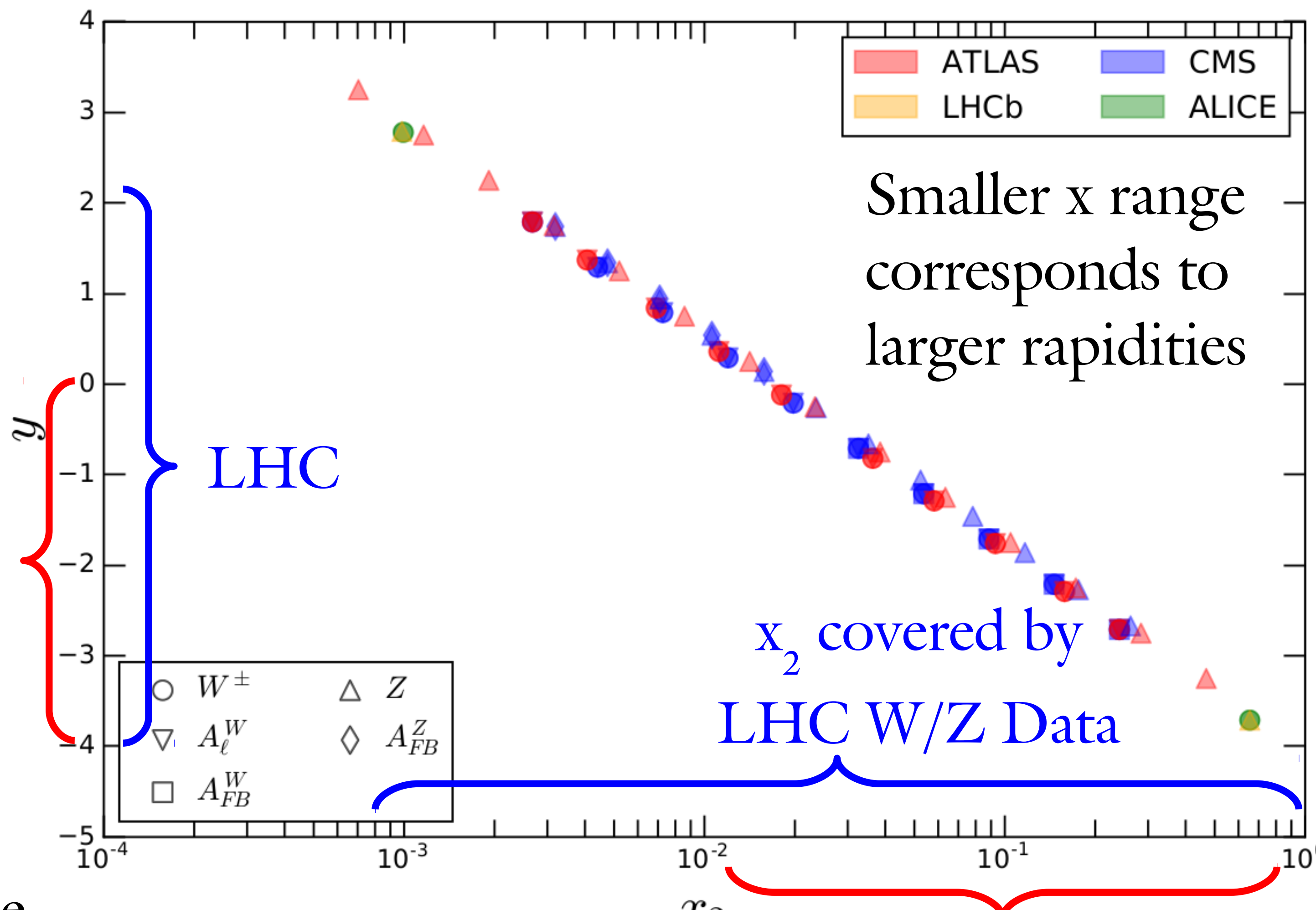
$$\sqrt{s_{pp}} = 7\ TeV \rightarrow \sqrt{s_{PbPb}} = 2.76\ TeV$$

$$\sqrt{s_{pp}} = 8\ TeV \rightarrow \sqrt{s_{pPb}} = 5.02\ TeV$$

pPb and PbPb collisions

- LHCb: $\sigma(Z \rightarrow \ell^+\ell^-)$
- ALICE: $\sigma(W^\pm \rightarrow \ell^\pm\nu)$
- ATLAS: $d\sigma(W^\pm \rightarrow \ell^\pm\nu)/dy$
  $d\sigma(Z \rightarrow \ell^+\ell^-)/dy$
  A_ℓ
- CMS: $d\sigma(W^\pm \rightarrow \ell^\pm\nu)/dy$
  $d\sigma(Z \rightarrow \ell^+\ell^-)/dy$
  A_ℓ
  A_{FB}

nCTEQ PDFs at the LHC



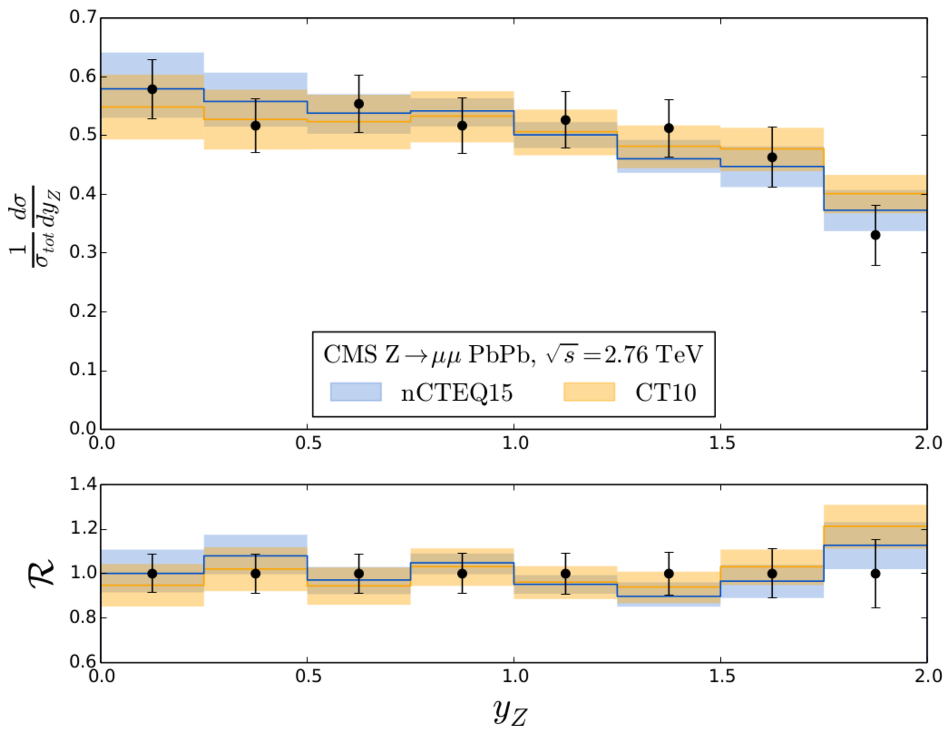
DIS

x_2 Covered by Existing DIS Data

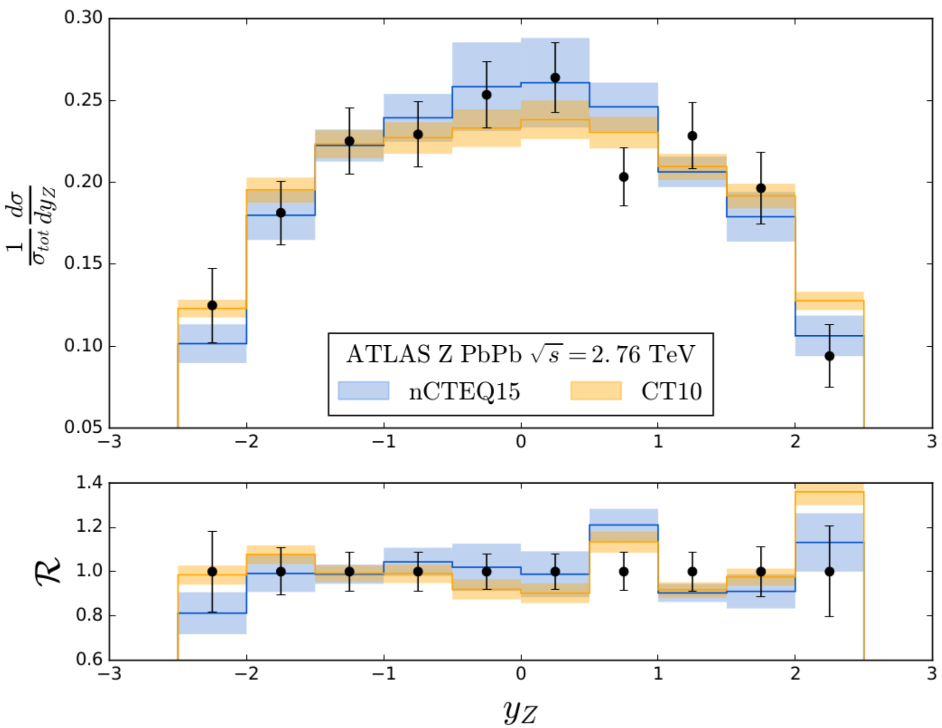
x_2 is the Lead PDF

W/Z Production

CMS PbPb Z



ATLAS PbPb Z



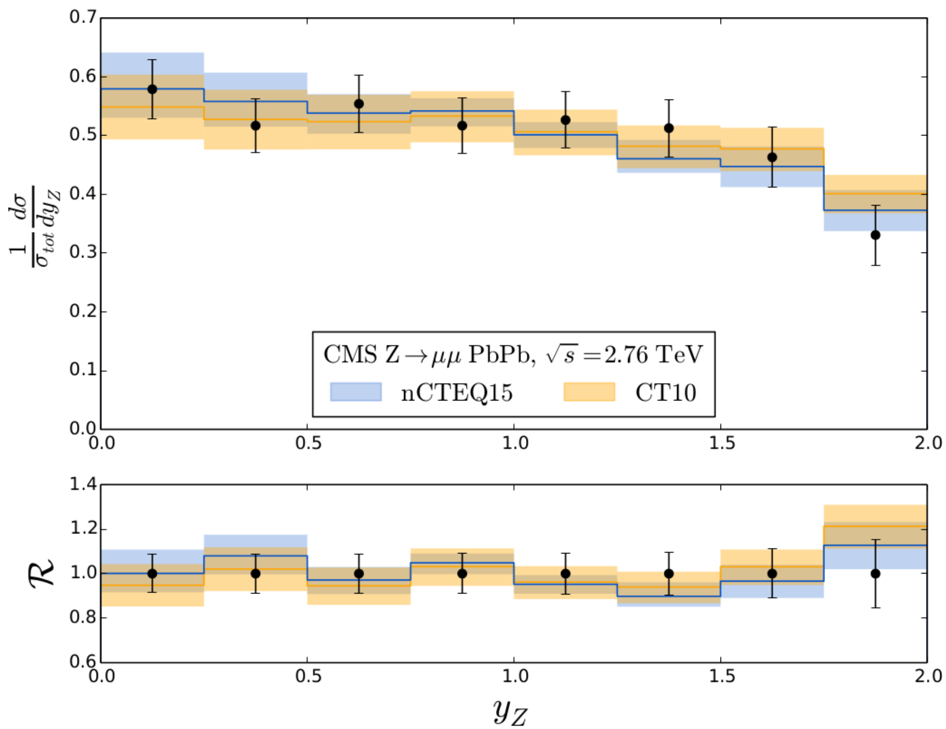
All predictions shown at NLO

Done in modified FEWZ software which allows for pp, pPb, and PbPb collisions

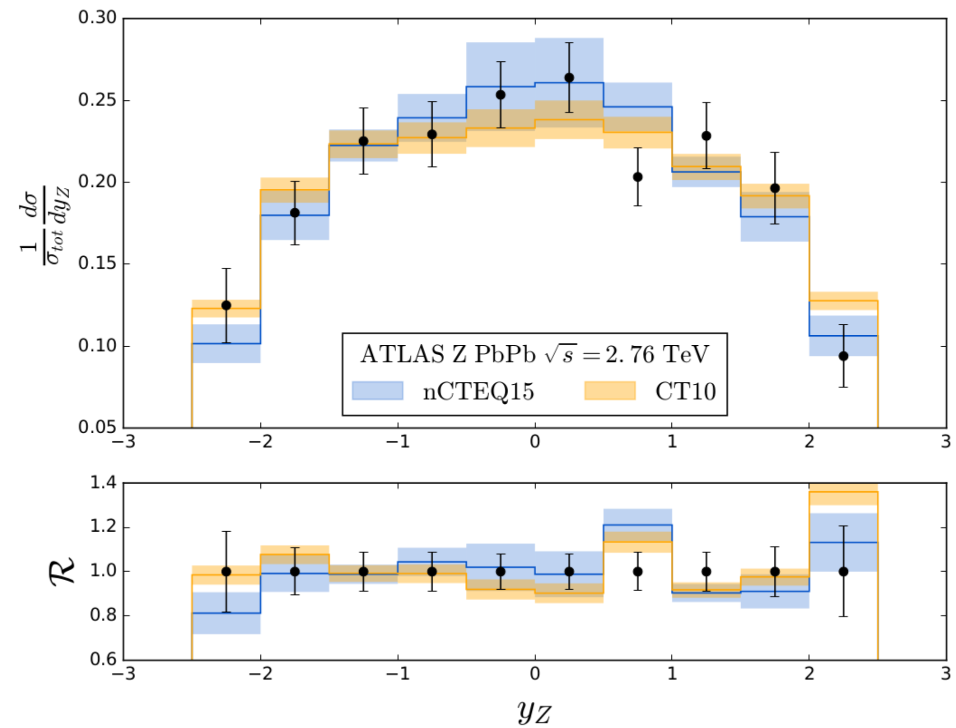
CT10 nucleus constructed using CT10 free protons

W/Z Production

CMS PbPb Z



ATLAS PbPb Z

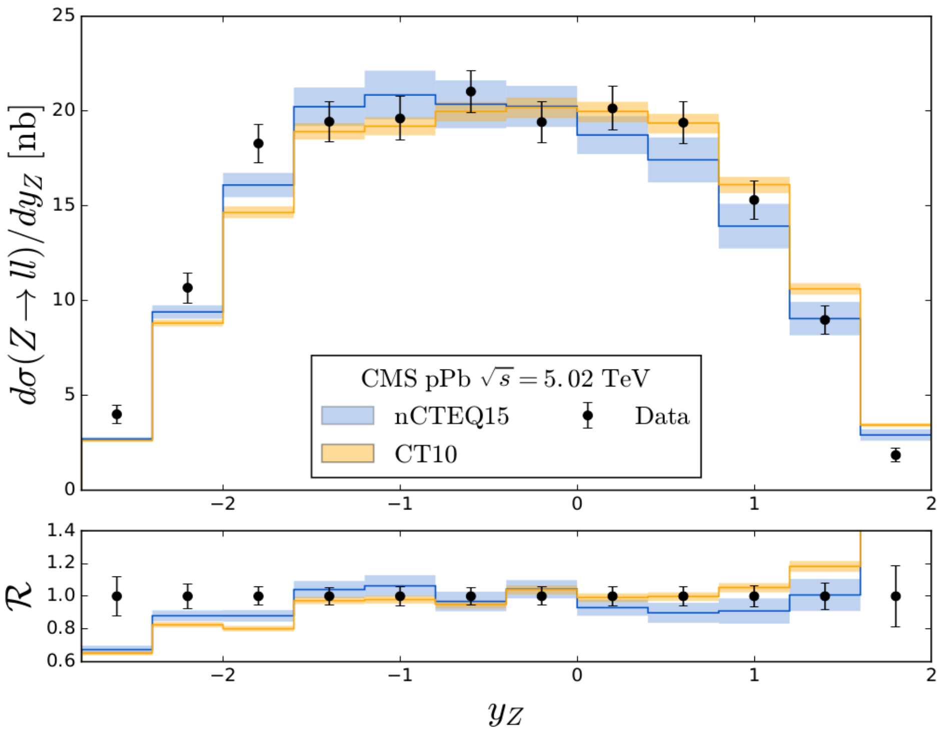


Good agreement between data and both sets

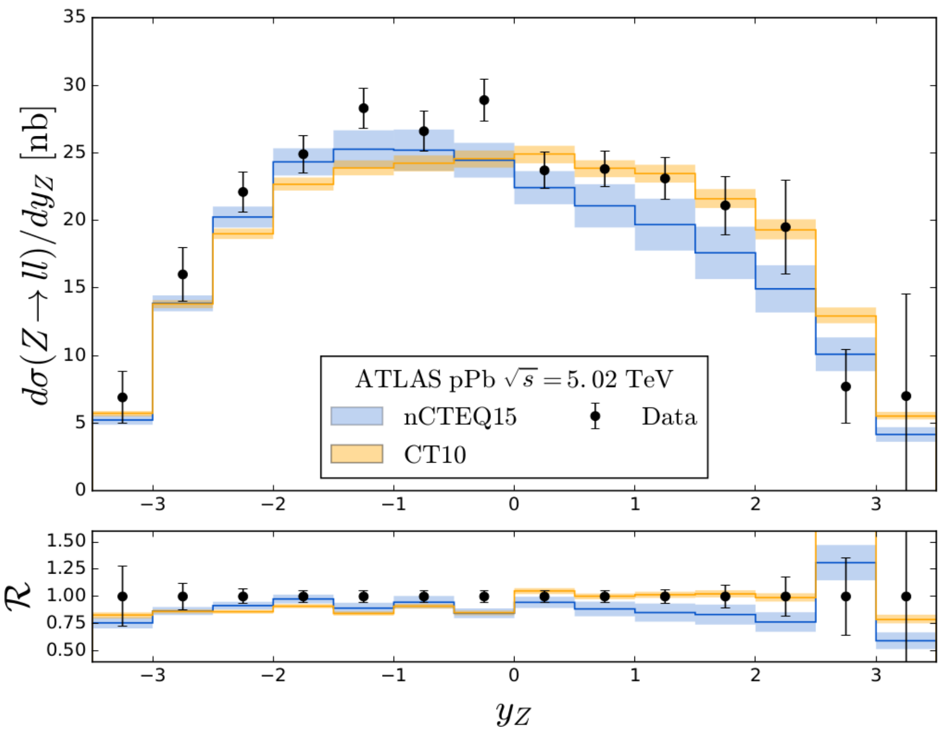
Not sensitive to nuclear corrections

W/Z Production

CMS pPb Z

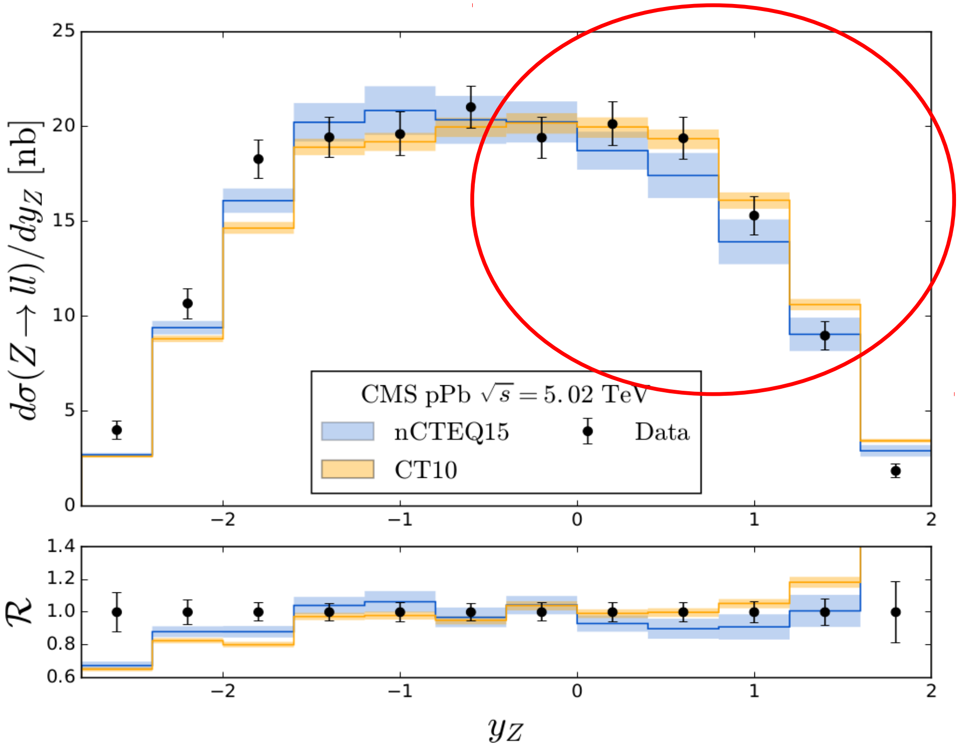


ATLAS pPb Z

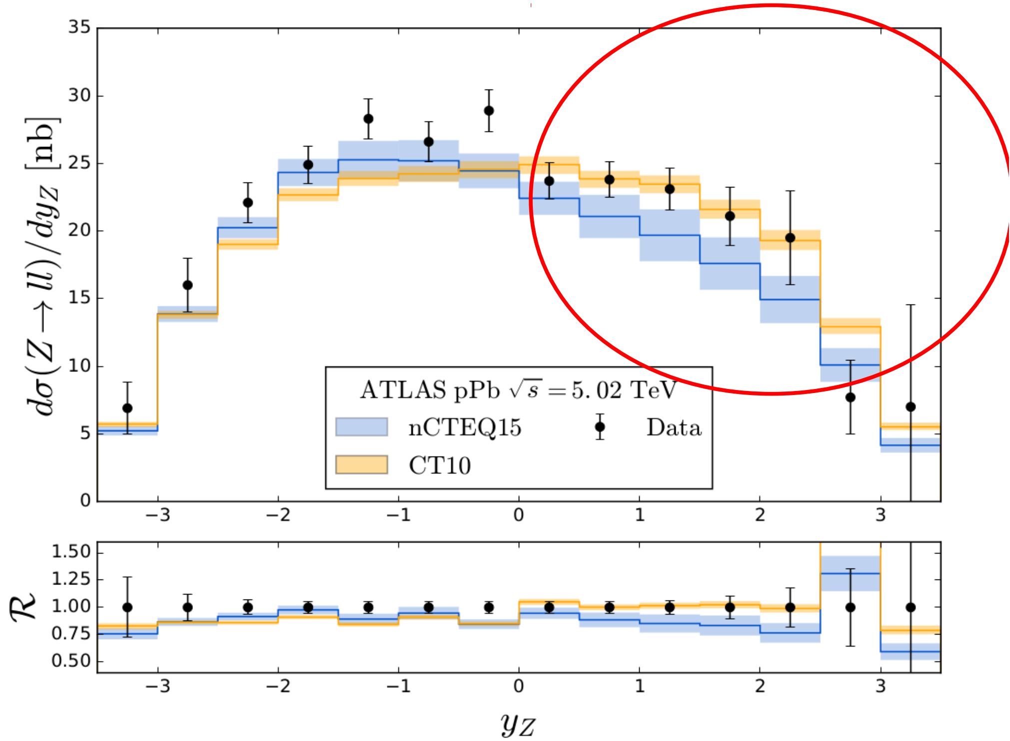


W/Z Production

CMS pPb Z



ATLAS pPb Z

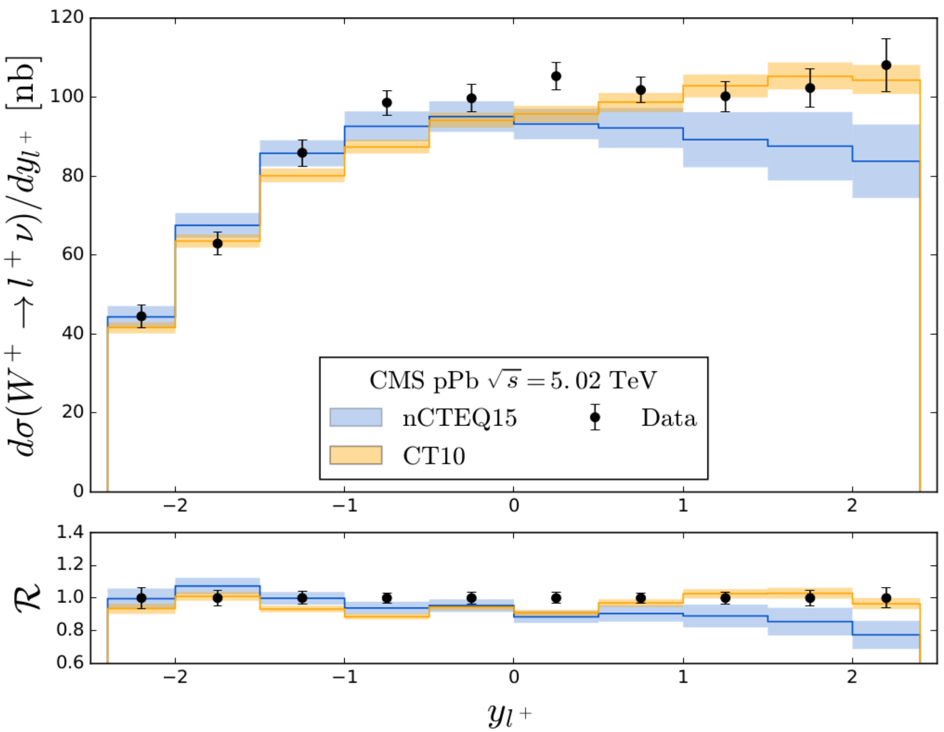


Slight deviance between data and CT10 from nCTEQ15

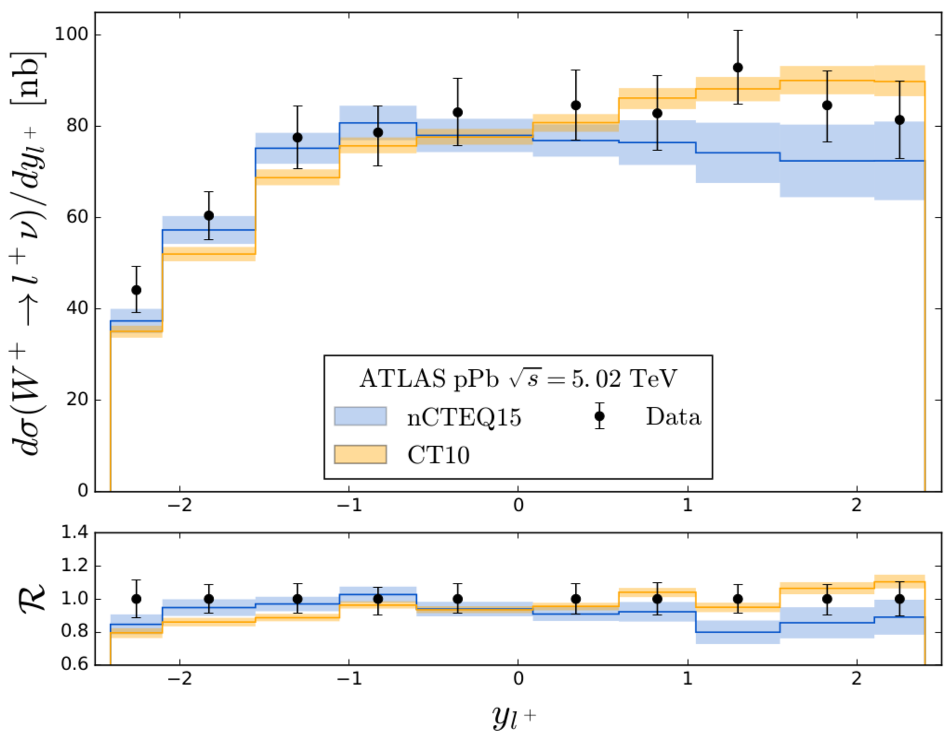
Good agreement for negative rapidities, the region we have data to constrain the PDF

W/Z Production

CMS pPb W+

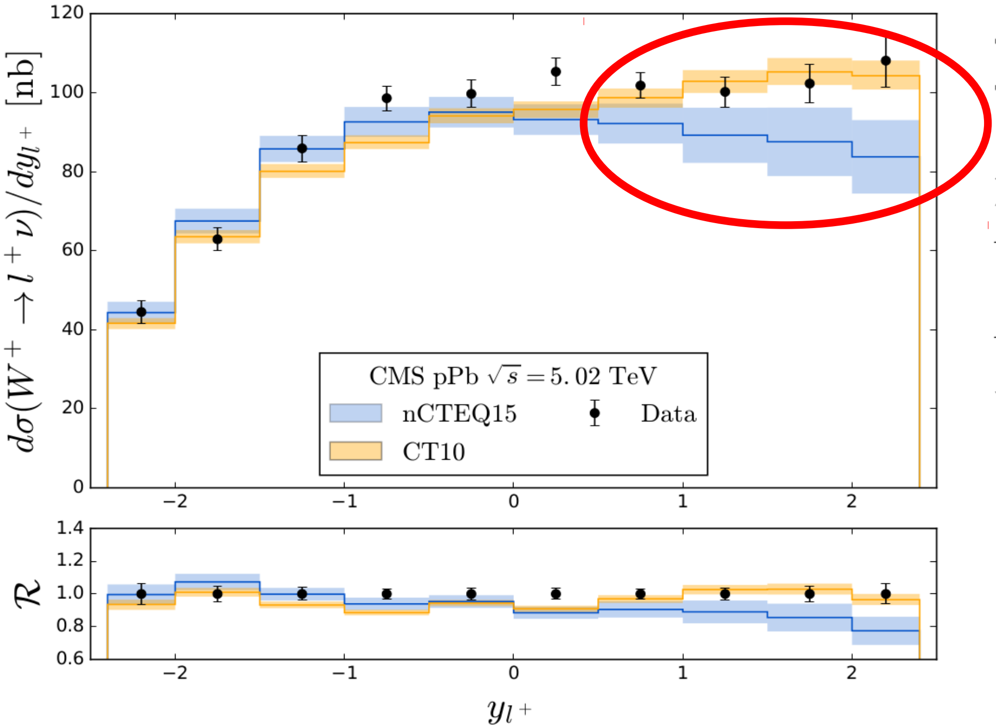


ATLAS pPb W+

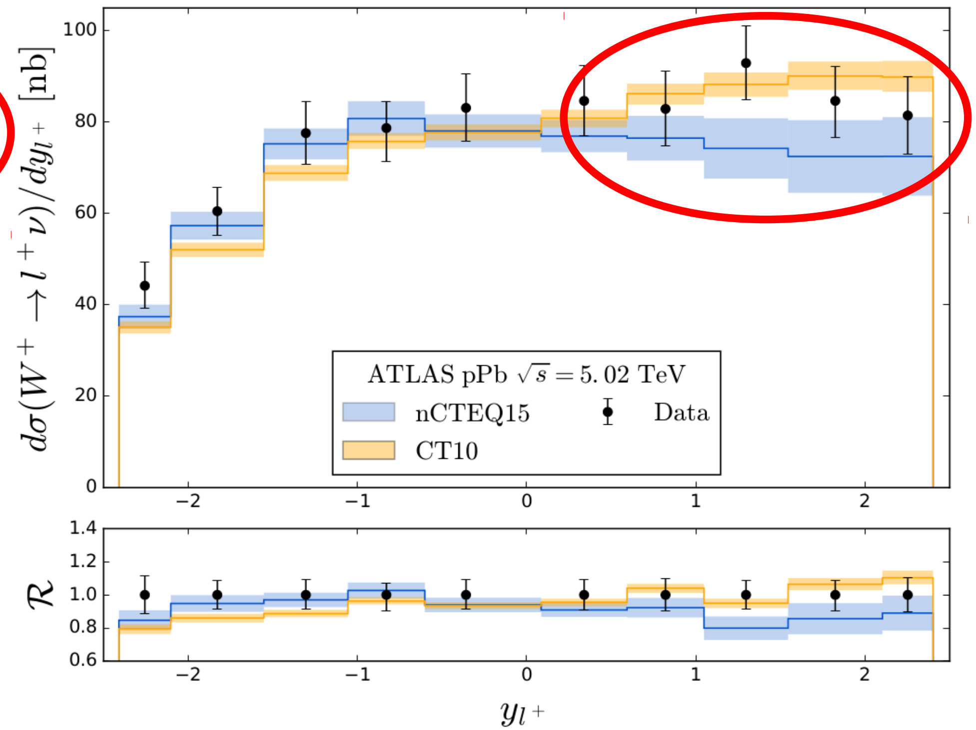


W/Z Production

CMS pPb W+



ATLAS pPb W+



Definite separation between data and CT10 from nCTEQ15

Indicates this data could be useful in constraining PDFs in this region

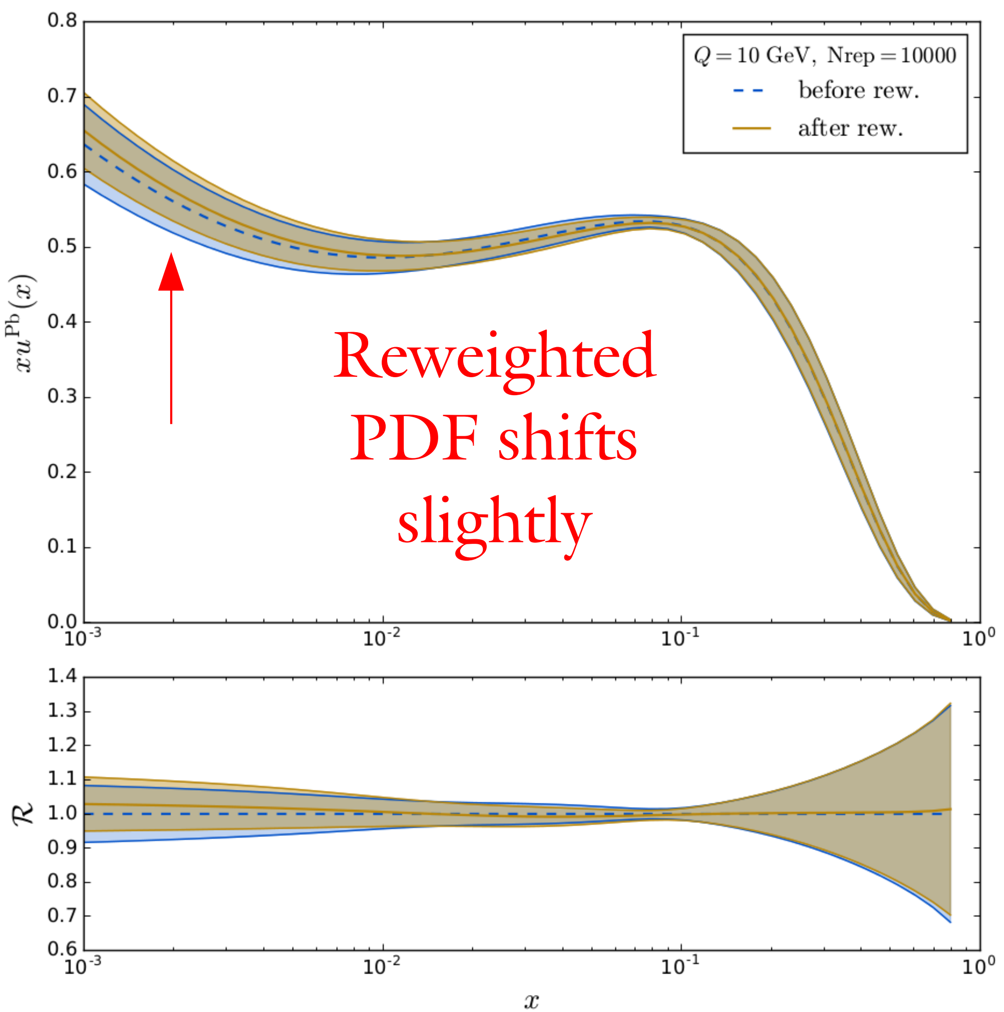
Reweighting

Technique using Bayesian Statistics to shift existing PDF

- Allows for new data to be added
  - DOES NOT REFIT

Can suggest the impact data might have on a future fit

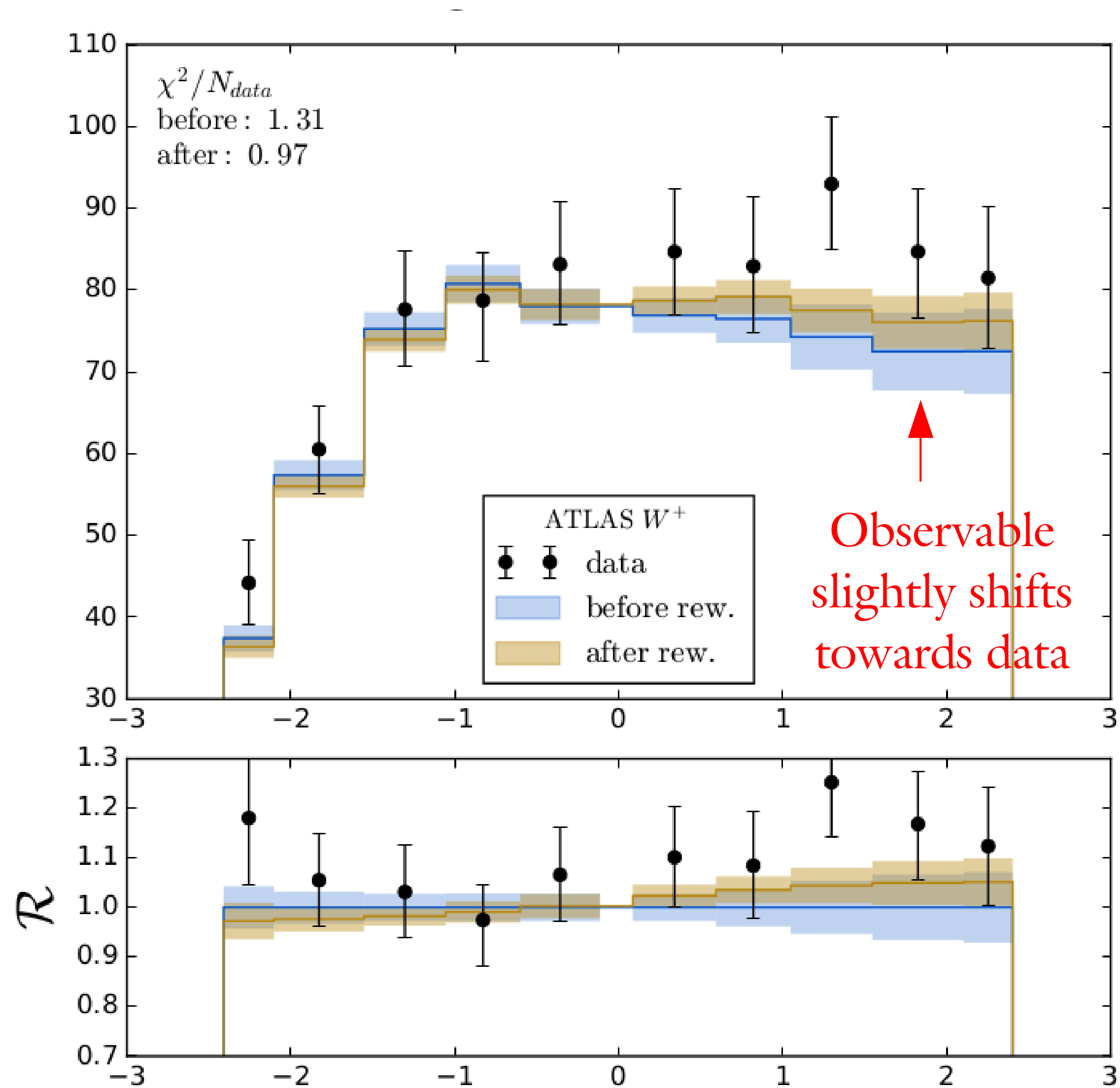
Limited to existing parameterization



Reweighting

Results indicated parameterization is inflexible and refit was needed

- Extrapolation in x_2 region corresponding to positive rapidity
  - Previously lacked data in this range
- Strange quark parameter could be opened
  - Currently fixed to up and down quarks



Refitting nCTEQ15

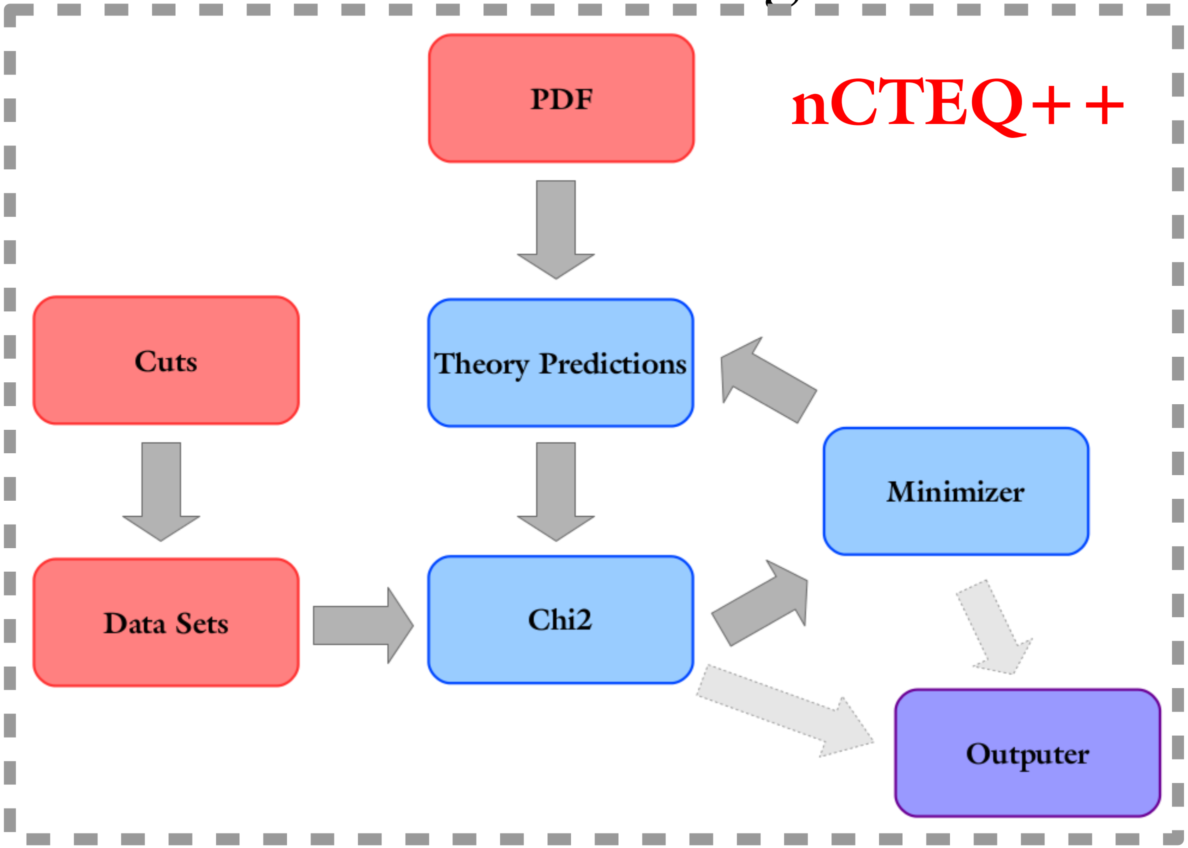
nCTEQ Status After Reweighting

- FORTRAN fitting code
  - Internal theory calculations
    - Process specific modules
  - Internal PDF evolution
- Large χ^2 from reweighting study
  - Poor description of the shape of LHC data
    - Particularly for positive rapidities
- No LHC data included
  - New theory module would be needed

nCTEQ++

What is nCTEQ++?

- A complete rewrite of the original nCTEQ FORTRAN fitting code in C++
- Changed the code to allow for modules when building a PDF
    - Evolution
    - Interpolation
    - Parameterization
- Fitting using Minuit
- Minor bug fixes



Validation: α_S

$$\frac{d\alpha_S}{d\ln(Q^2)} = \beta(\alpha_S(Q^2)) = -(b_0\alpha_S^2 + b_1\alpha_S^3 + b_2\alpha_S^4 + \ldots)$$

nCTEQ++: (HOPPET)

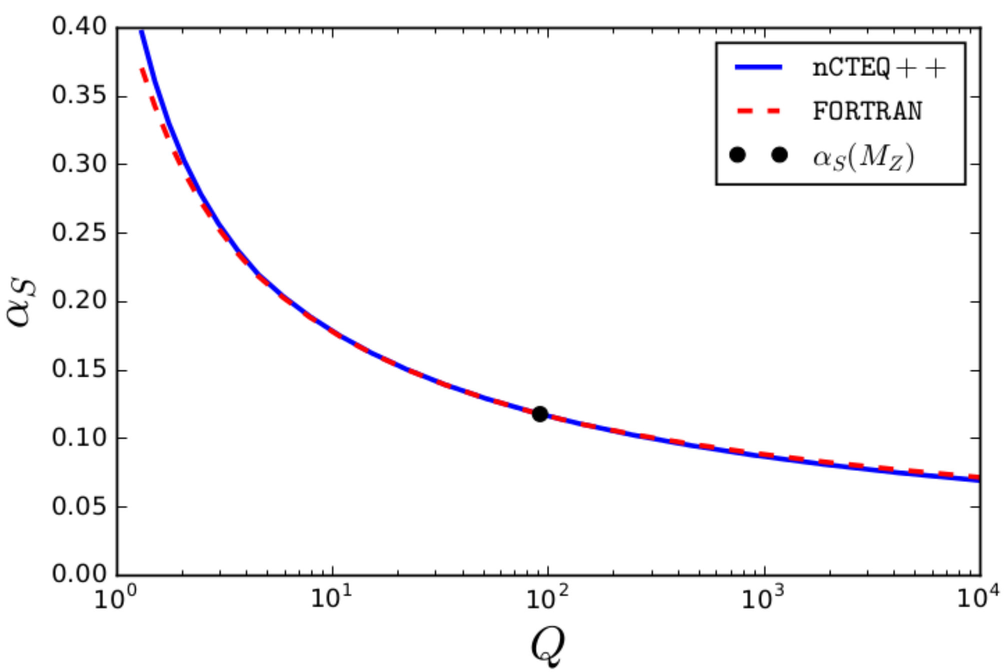
Runge-Kutta numerical solution

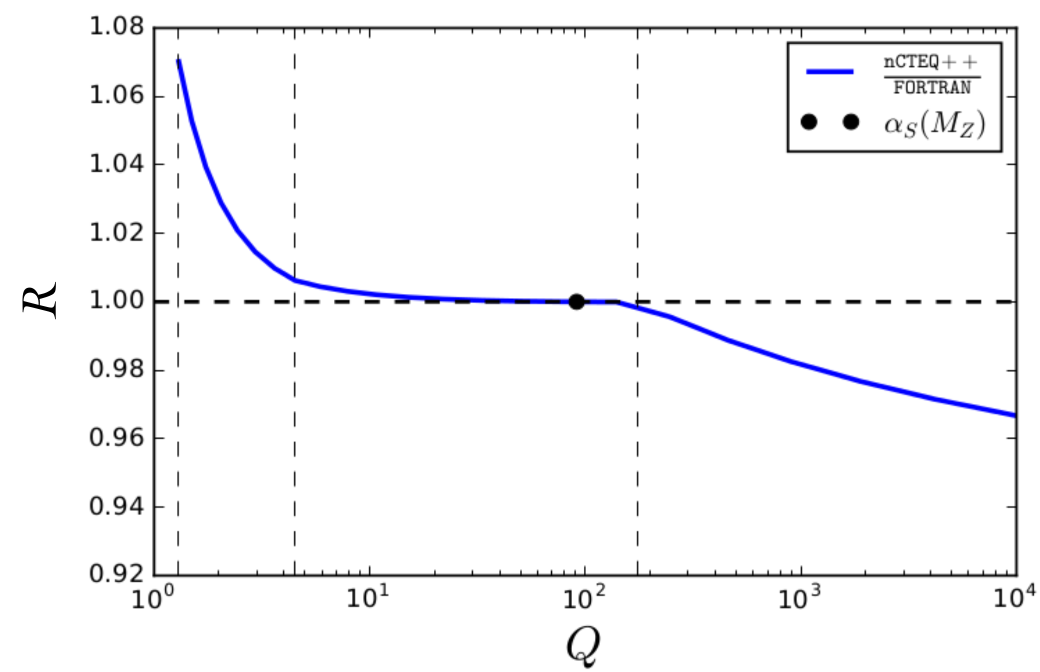
Number of Quark Flavors – 4, 5, 6

FORTRAN:

Truncated analytic series solution

Number of Quark Flavors – 4, 5





Validation: Evolution

HOPPET to provide PDF evolution

- Accepted in PDF community (PDF4LHC)
- Externally maintained

PDFs match at Q_0

Differences in evolution arise from different α_S

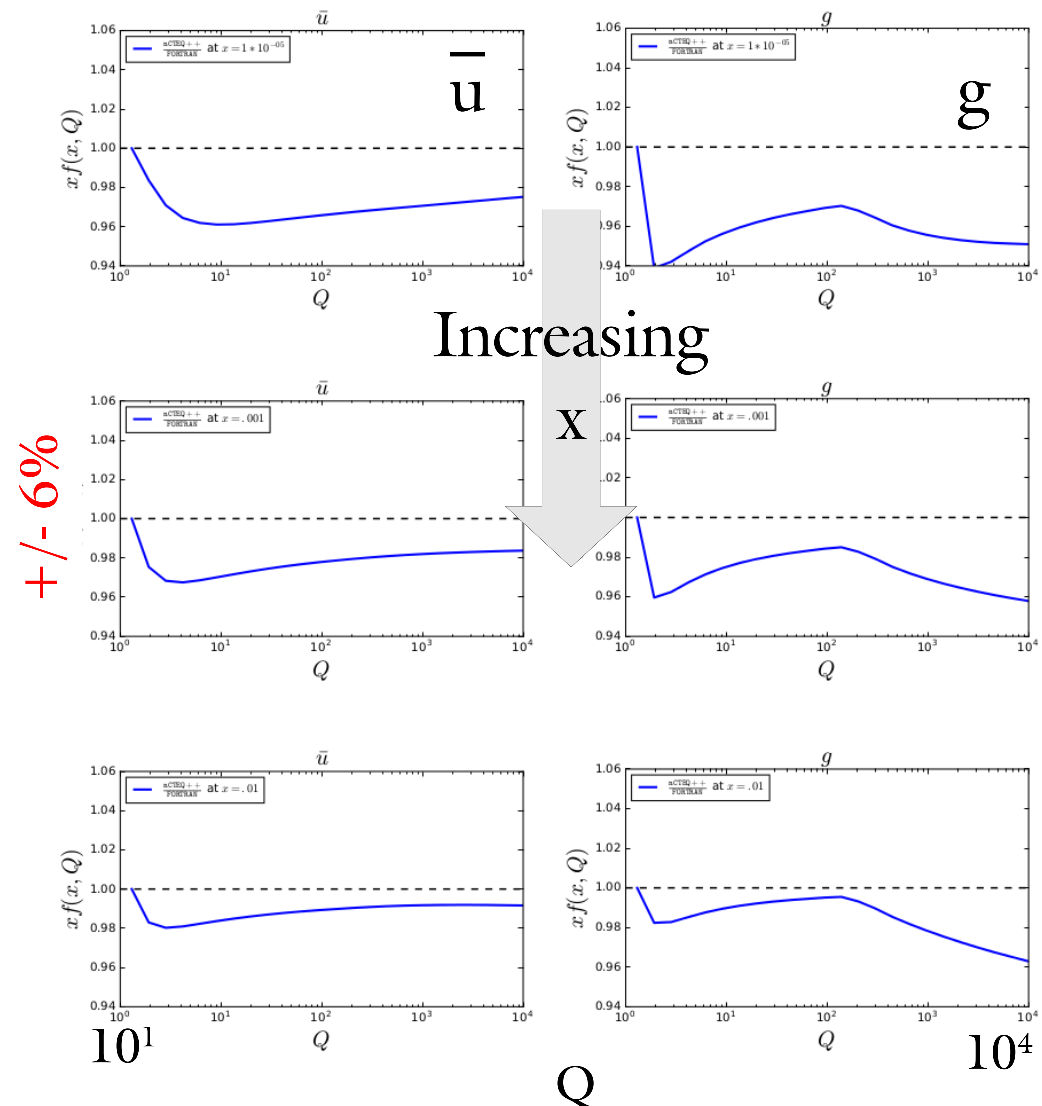
- HOPPET uses Runge-Kutta

Differences consistent across

Q range, x range, all flavors

Gluon reflects top quark threshold

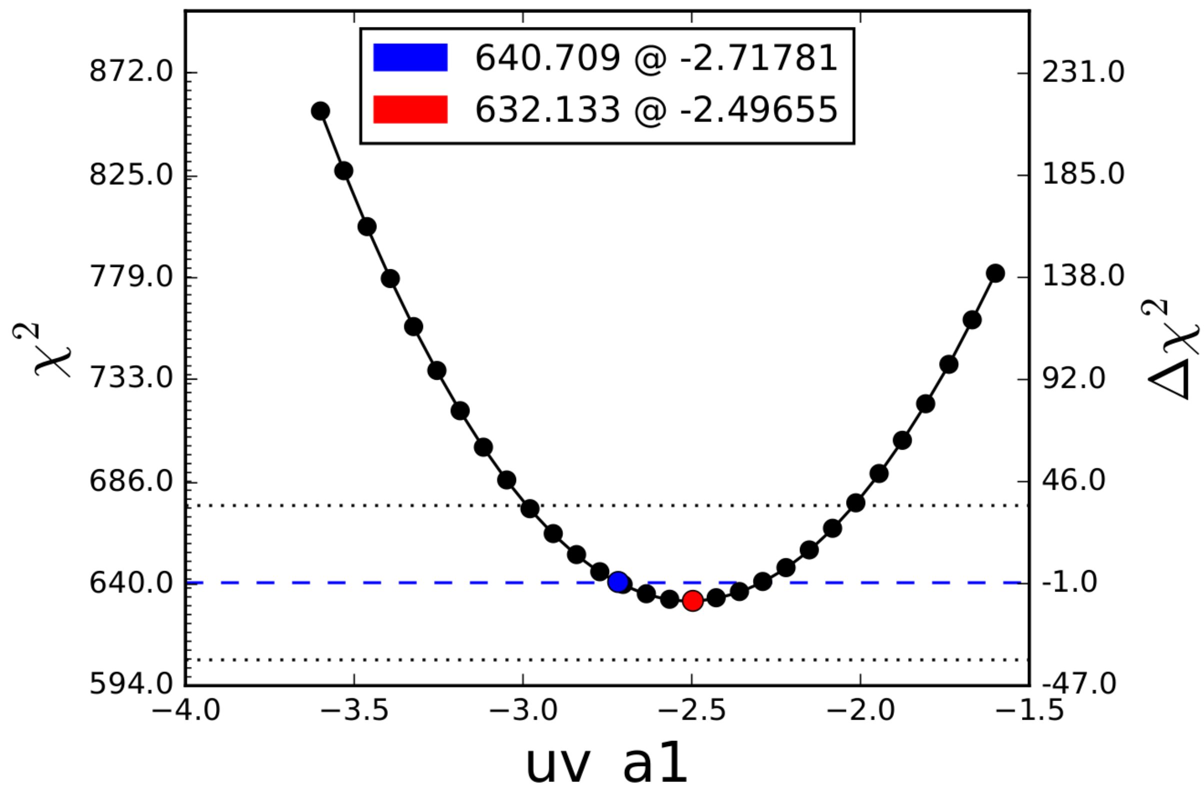
Ratio of PDF Evolution Codes



Validation: Parameter Scans

How did the χ^2 change for nCTEQ15 in nCTEQ++?

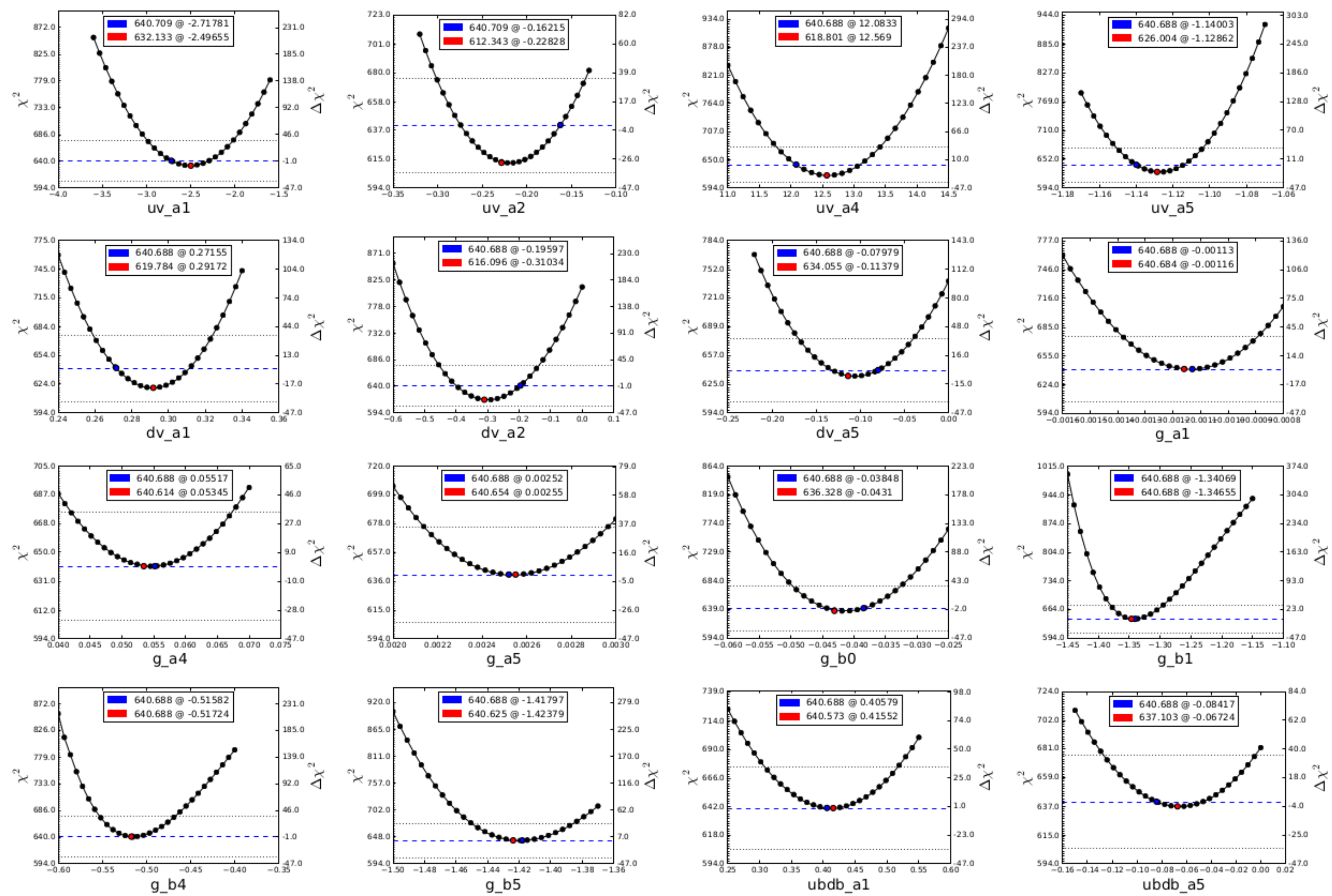
- Minimum might have changed in new code
  - Scan each parameter fit in nCTEQ15
    - Step through parameter space
      - Calculate χ^2 at each point
  - χ^2 Tolerance (t) = 35
- All parameters fell within tolerance



Validation: χ^2

How did the χ^2 change for nCTEQ15np in nCTEQ++?

- Original:
  - χ^2 : 625.6
  - 708 points
  - 0.883 χ^2/d.o.f.
- nCTEQ++:
  - X^2 : 640.7
  - 708 points
  - 0.905 χ^2/d.o.f.
- $\Delta\chi^2$: 15.1
  - Less than t=35



Scans for the 16 parameters fit in nCTEQ15np

Theory Predictions

Goal: Include new LHC data in nCTEQ15 PDF fit

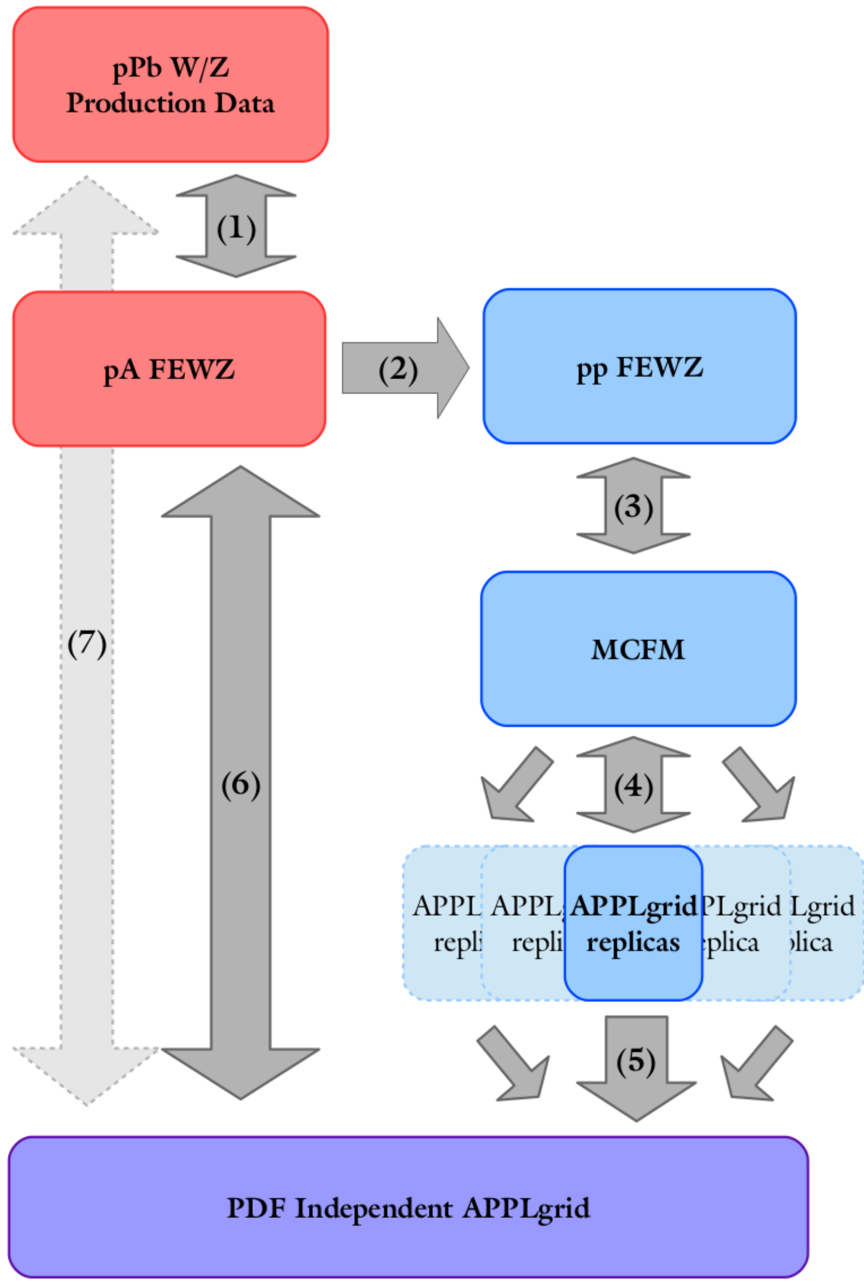
- Theory predictions are very slow and time consuming
  - Grid techniques drastically speed up the process
- Theory code must be tuned to match experimental measurement
- Theory codes available:
  - pAFEWZ
    - Modified to allow for pp, AA, pA modes
    - Previously tuned in reweighting study to match experiments
    - No grid techniques available
  - MCFM
    - Can only run in symmetric pp or AA modes
    - Not tuned to match experimental measurements
    - Links directly to APPLGrid
    - Extensive library of processes

Gridded Theory Predictions

- Produced in APPLGrid via MCFM
  - Cut-dependent arrays in (*x*,*Q*,*flavor*) space
  - Filled with weighted matrix elements from Monte Carlo integration
  - Precalculated, interpolated and summed, reducing computation time
  - Grids can be PDF-independent with enough statistics

$$\underbrace{\hat{\sigma}(x,Q)}_{\text{Slow matrix element}} \rightarrow \underbrace{\hat{\sigma}_i(x_i,Q_i)}_{\text{Fast Interpolated Grid}}$$

Including New Theory



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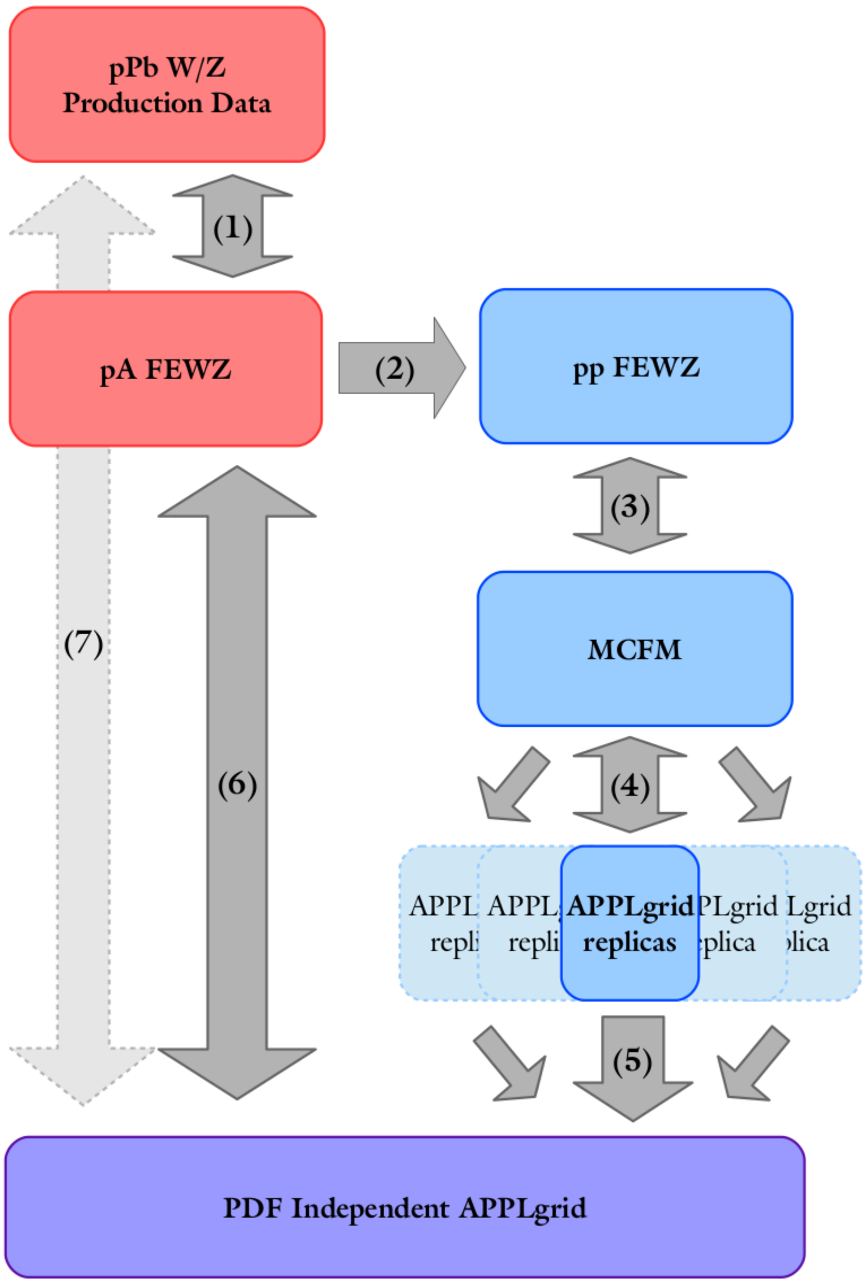
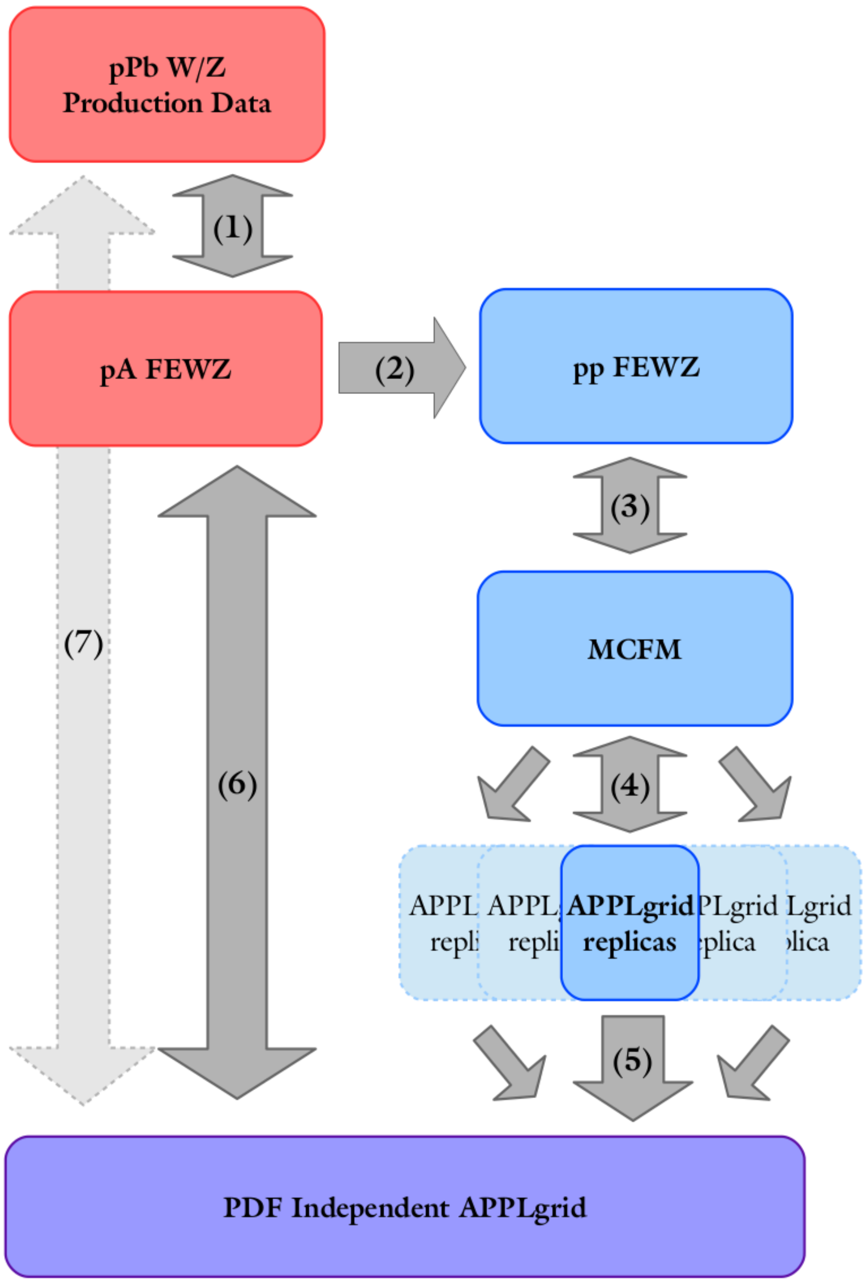


(1) Data matched to pAFEWZ in reweighting

Including New Theory

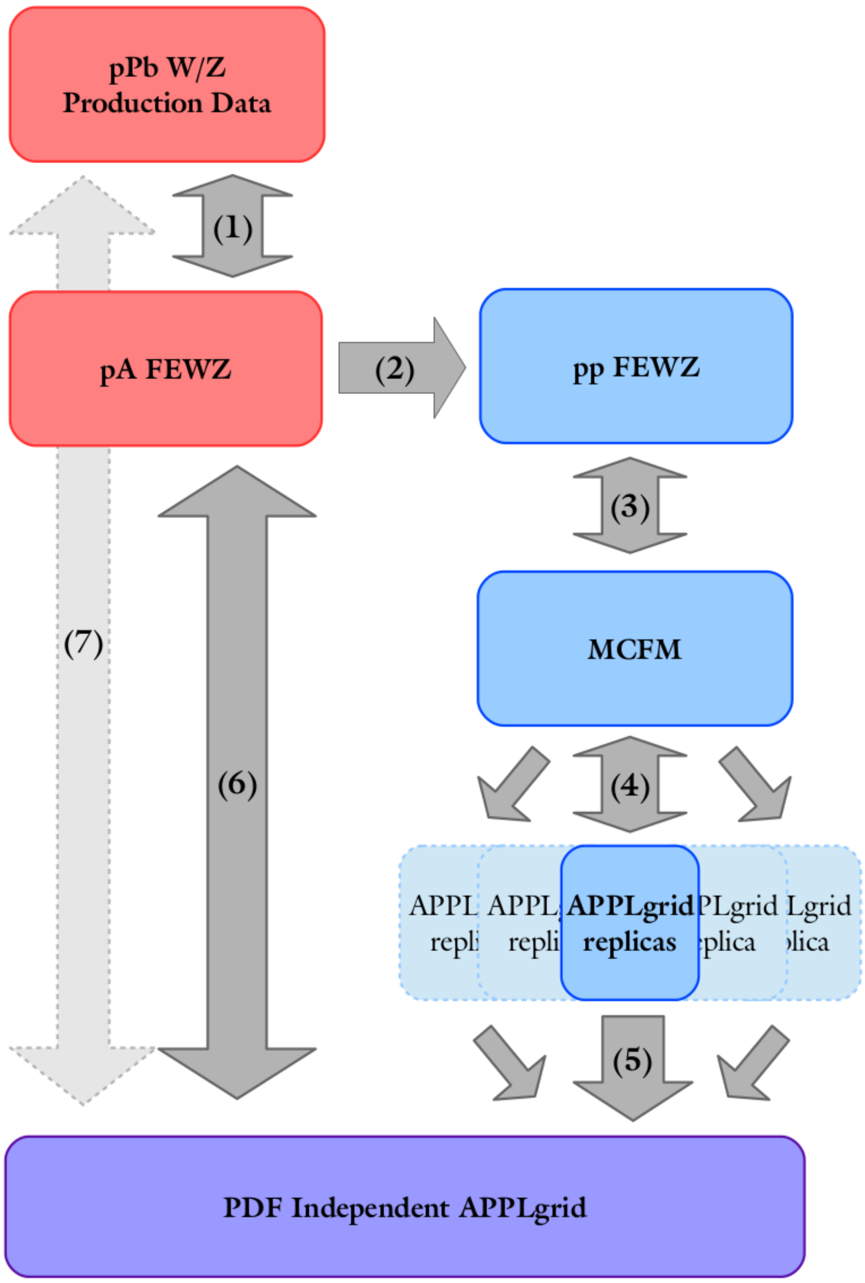


(1) Data matched to pAFEWZ in reweighting

(2) Run FEWZ in symmetric pp - mode

Maintains cuts and binning from asymmetric mode

Including New Theory



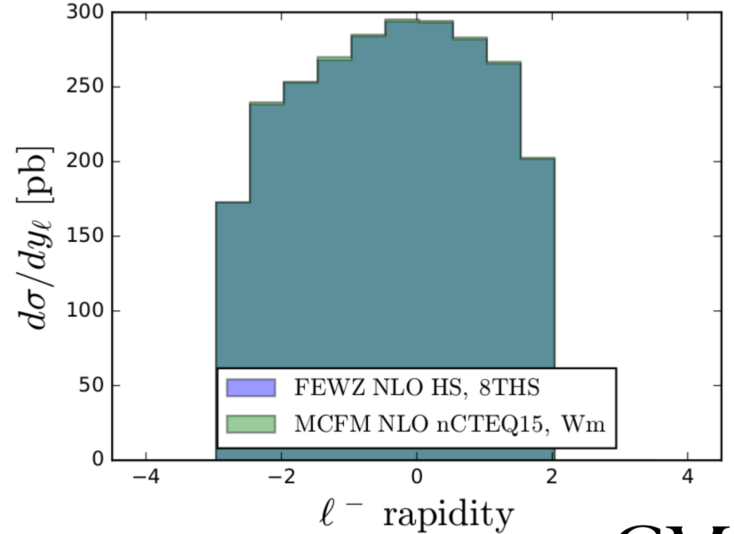
(1) Data matched to pAFEWZ in reweighting

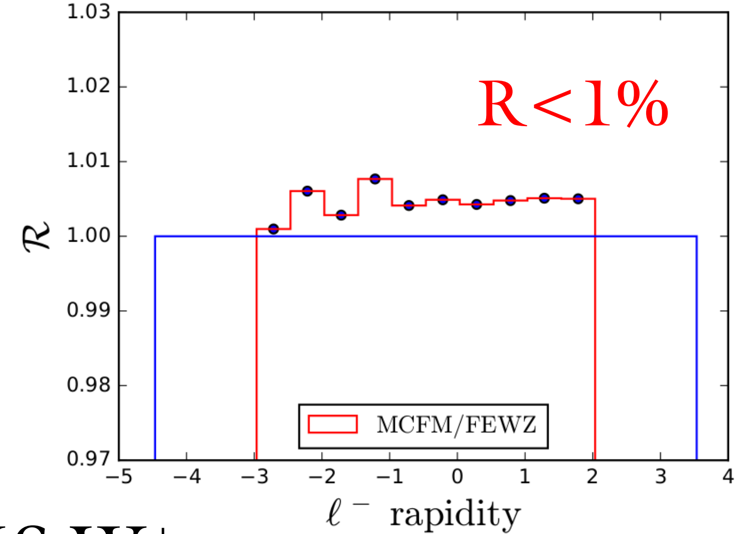
(2) Run FEWZ in symmetric pp - mode

(3) Compare pp FEWZ to pp MCFM

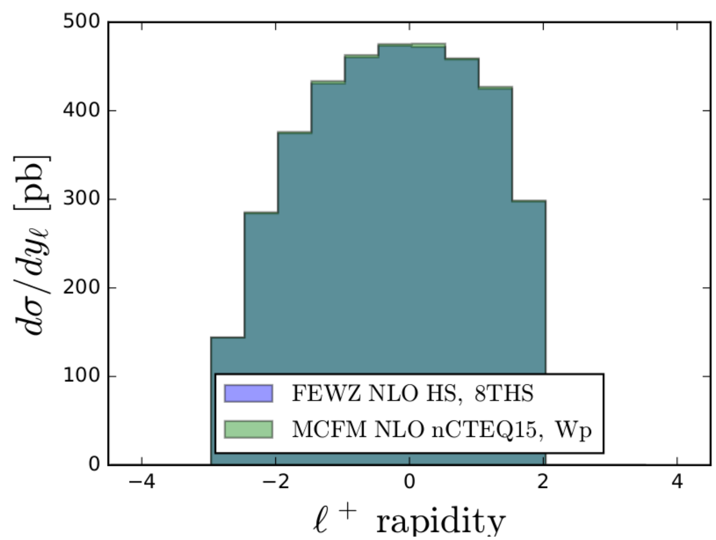
Compare FEWZ-pp to MCFM-pp

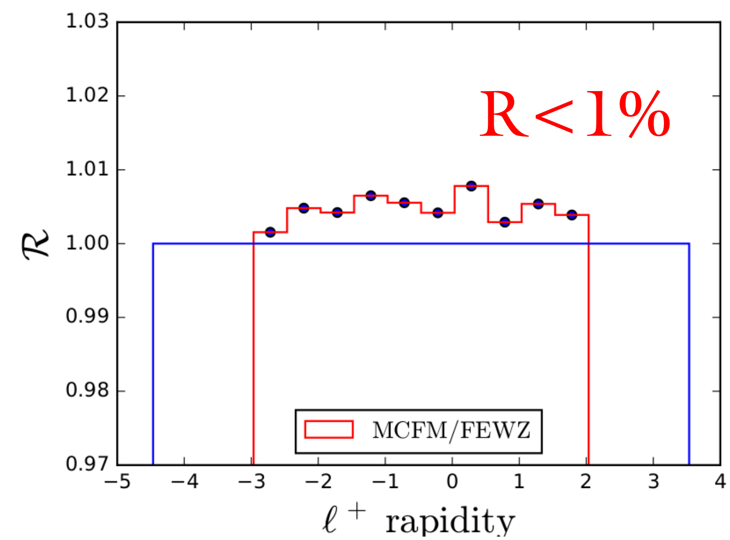
CMS W-



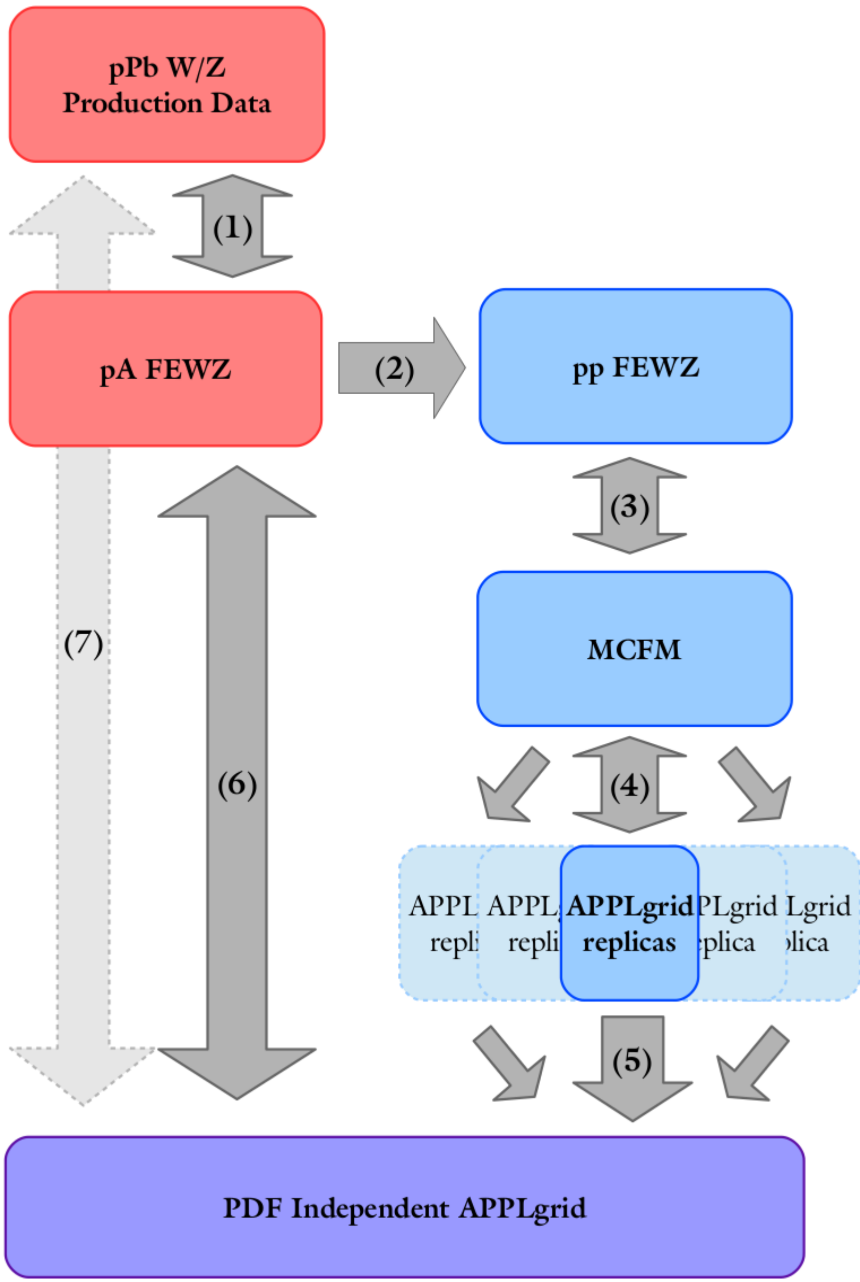


CMS W+





Including New Theory



(1) Data matched to pAFEWZ in reweighting

(2) Run FEWZ in symmetric pp - mode

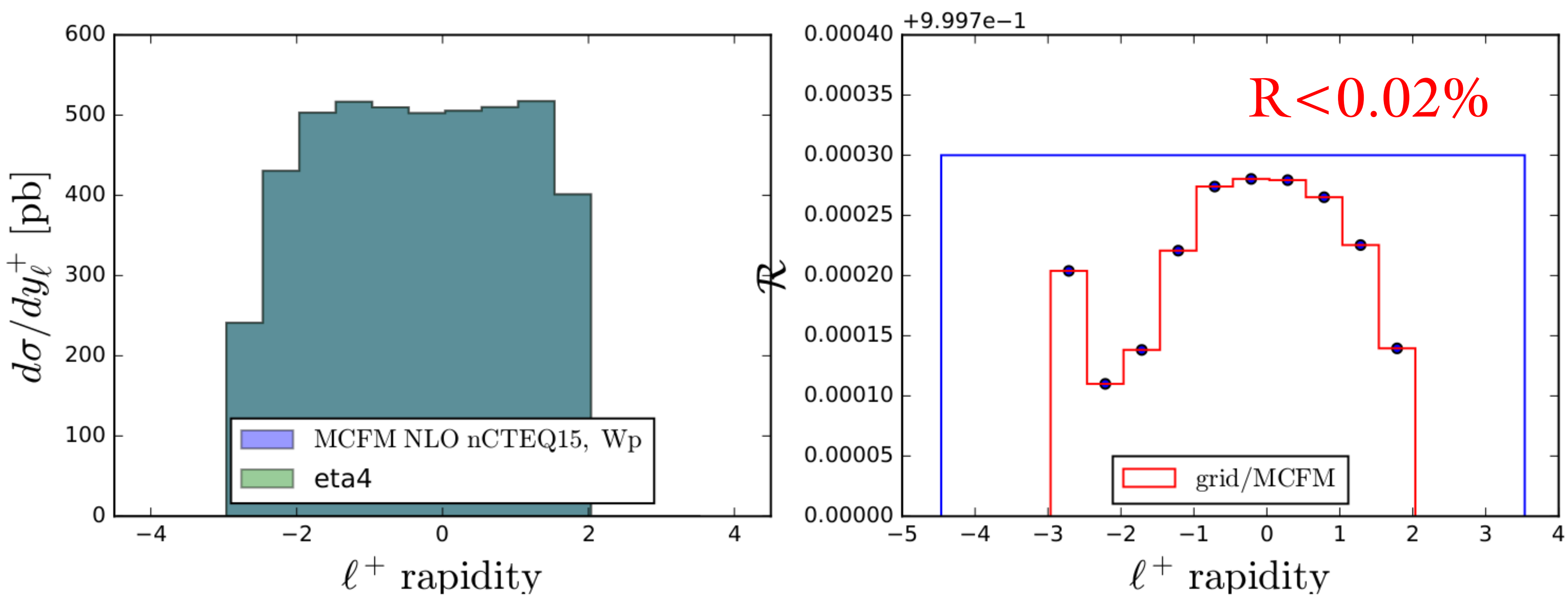
(3) Compare pp FEWZ to pp MCFM

(4) Generate APPLgrid grids

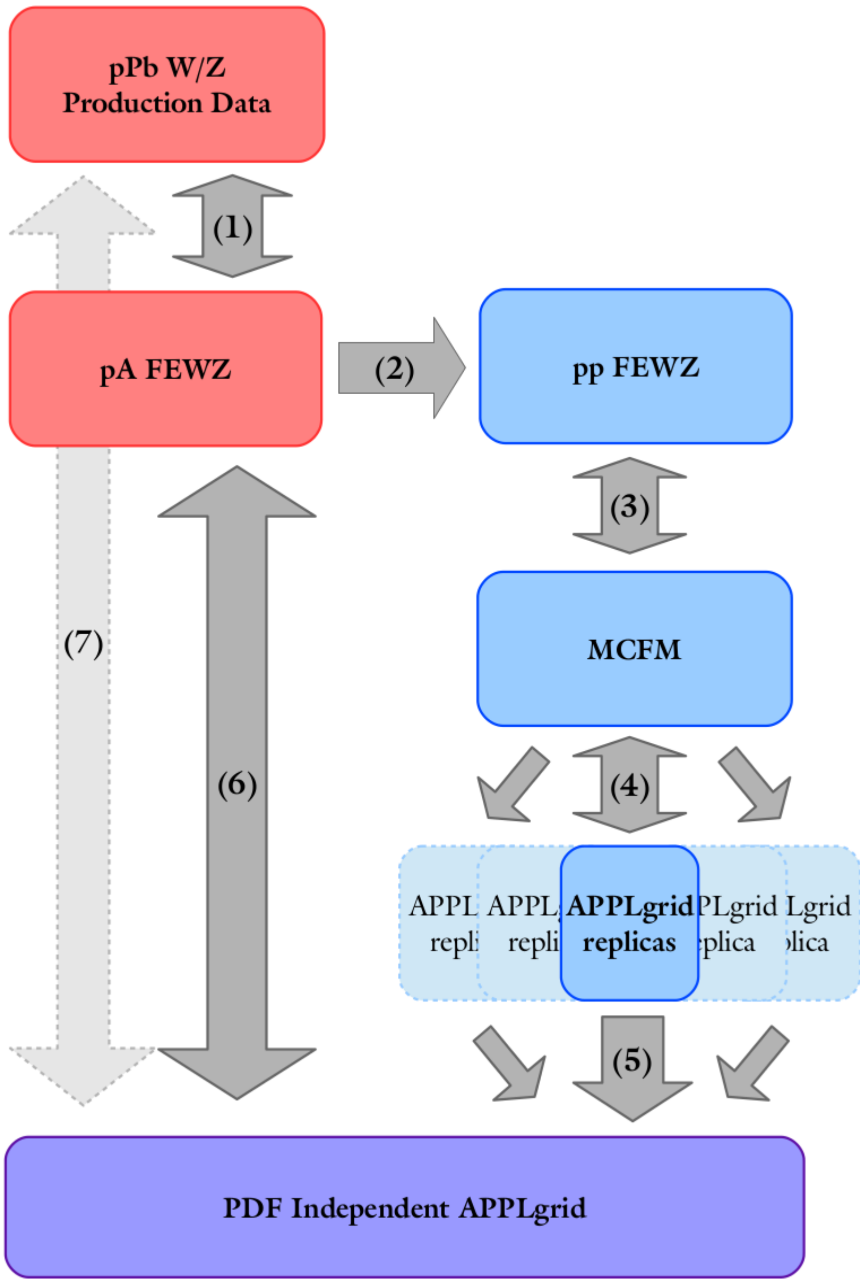
- Using mcfm-bridge
- Different Monte Carlo seeds

Producing grids using MCFM

- Events generated in MCFM
  - Extracted with mcfm-bridge
  - Stored in grids by APPLgrid
- Grids then convoluted with PDF used to generate events
- Different Monte Carlo seeds for MCFM change the events and subsequently the grids
  - ManeFrame allows this to run quickly in parallel



Including New Theory



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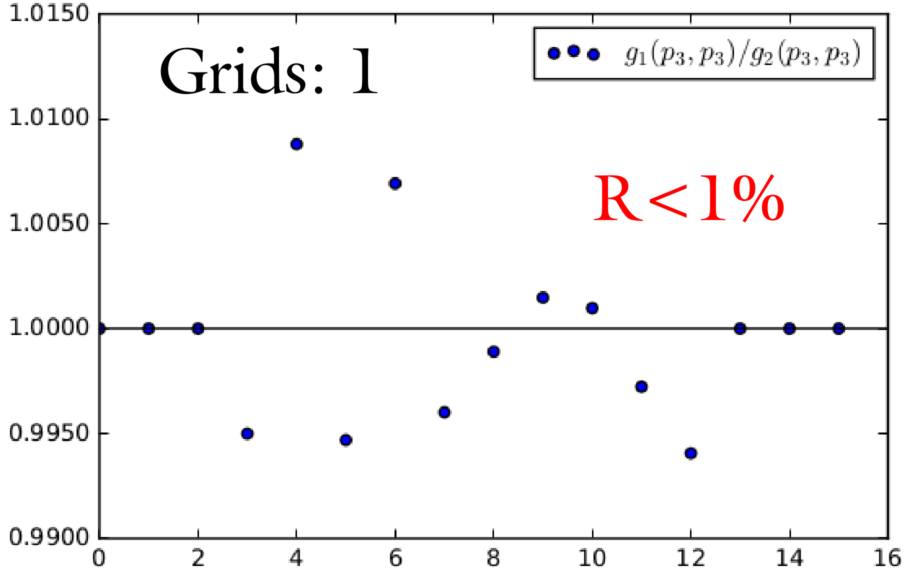
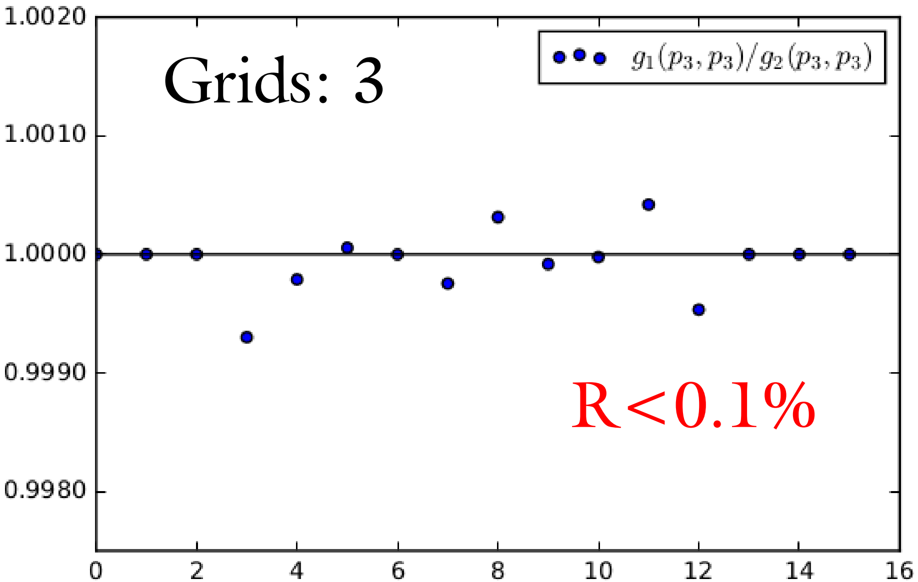
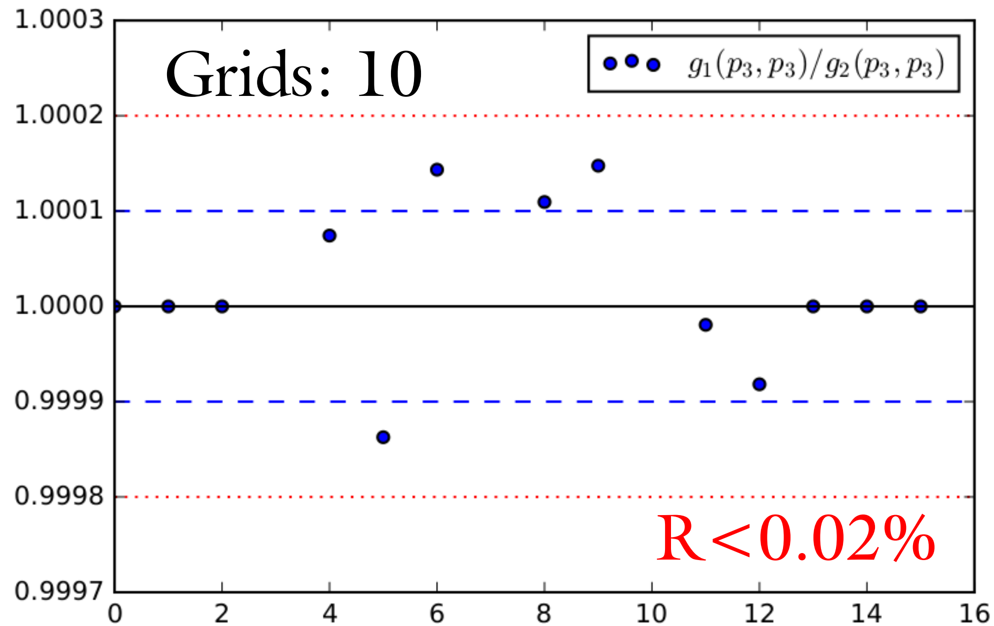
(5) Combine replica grids into a single PDF independent grid

PDF Independence for Grids

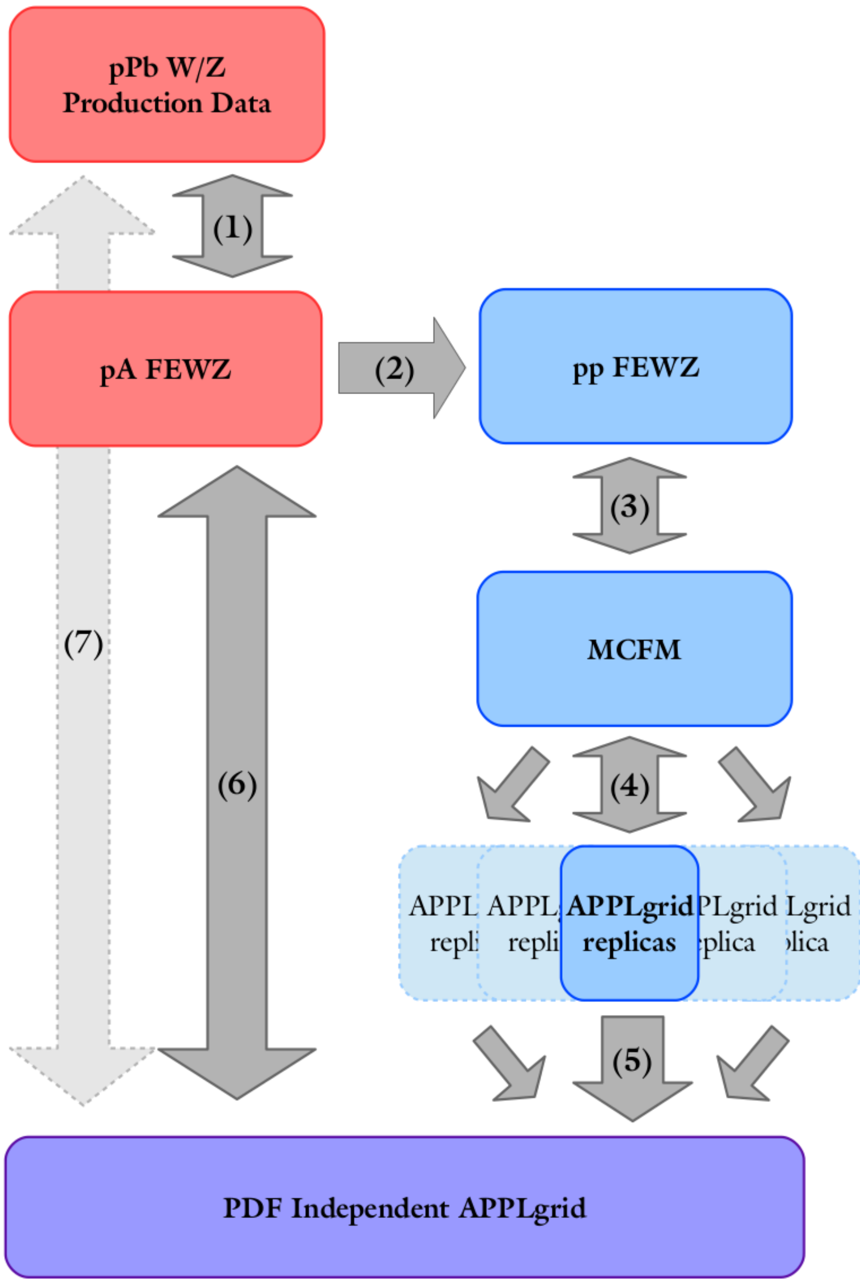




- Grids can be combined increase their statistics
  - applgrid-combine utility
  - Improve statistics
  - Decrease reliance on underlying PDF



Including New Theory



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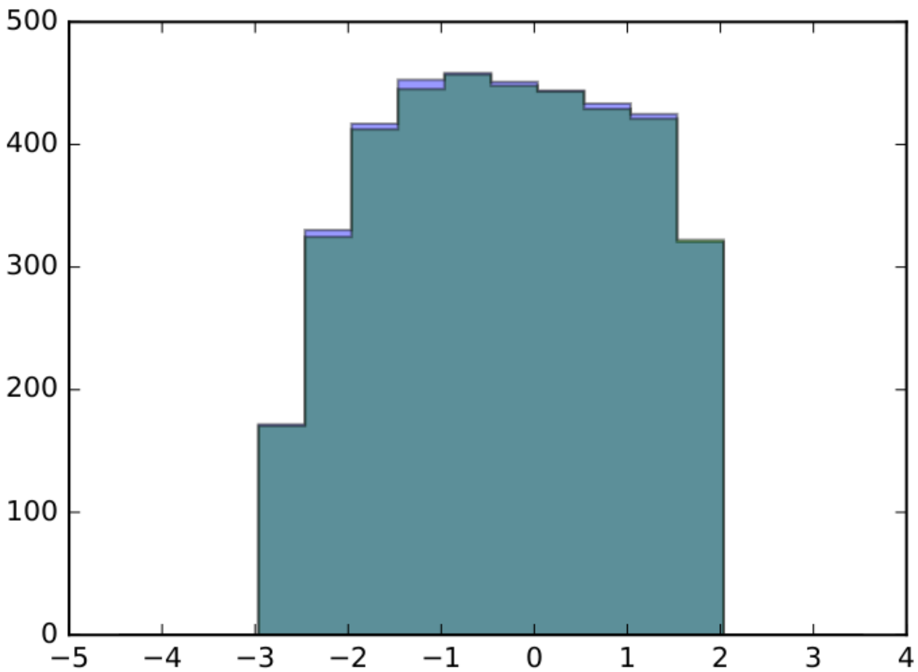
- Using mcfm-bridge
- Different Monte Carlo seeds

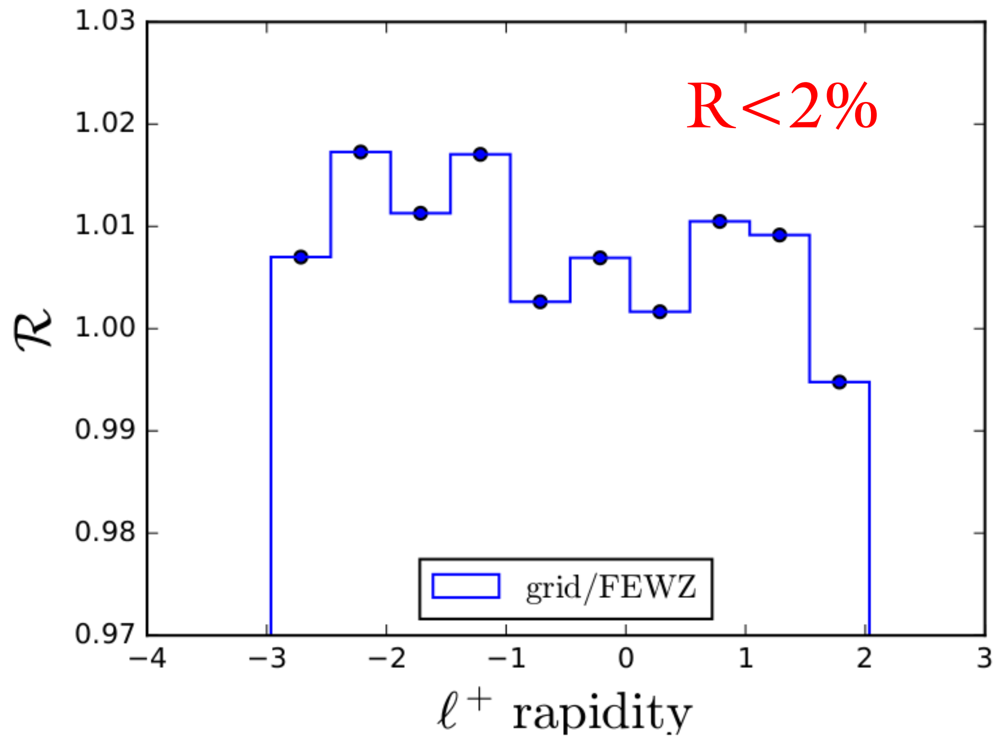
(5) Combine replica grids into a single PDF independent grid
- Using applgrid-combine

(6) Convolute PDF independent grid with asymmetric PDFs to compare to pAFEWZ

Comparing pAFEWZ to PDF Independent grid

- 10 grids from different MCFM runs combined into final grid
- Final grid then convoluted with pPb PDFs
  - Same that were used in pAFEWZ run





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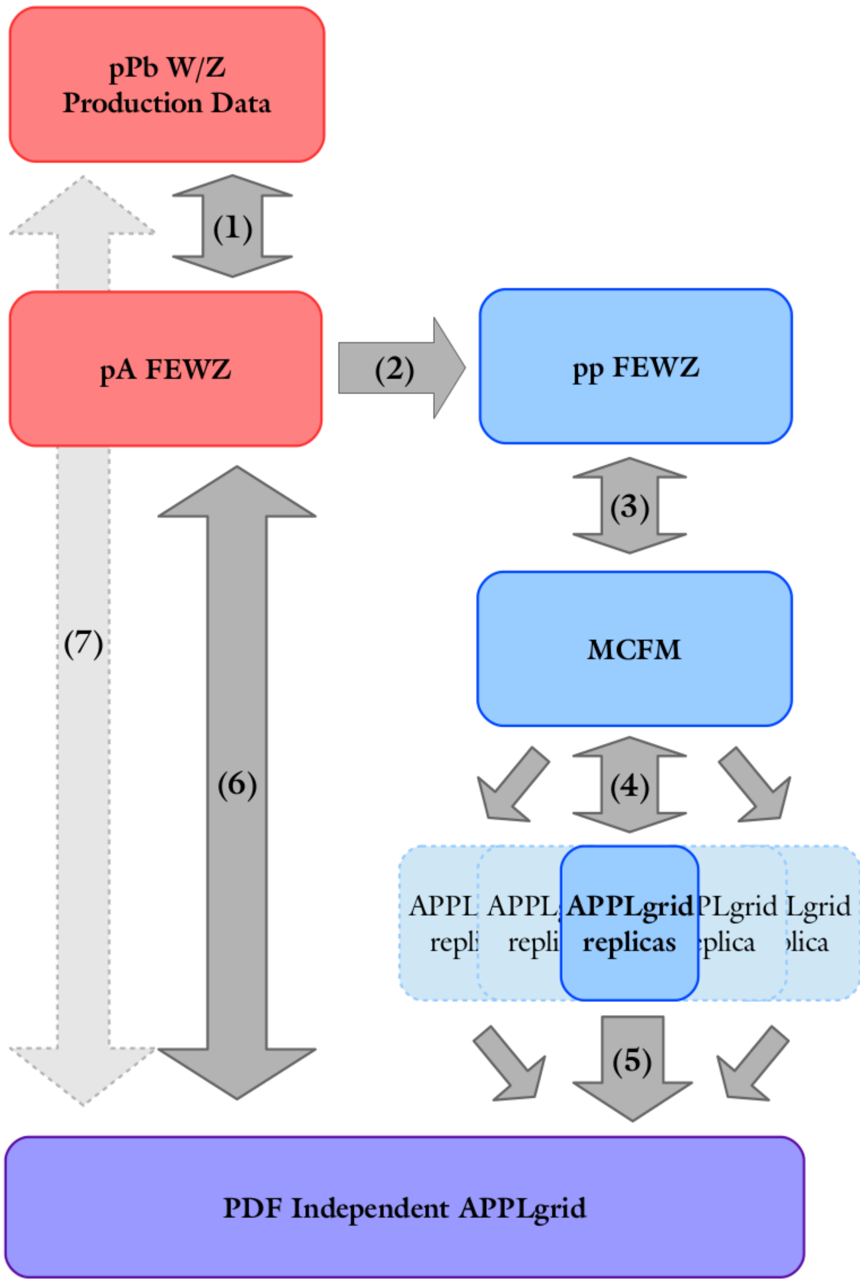

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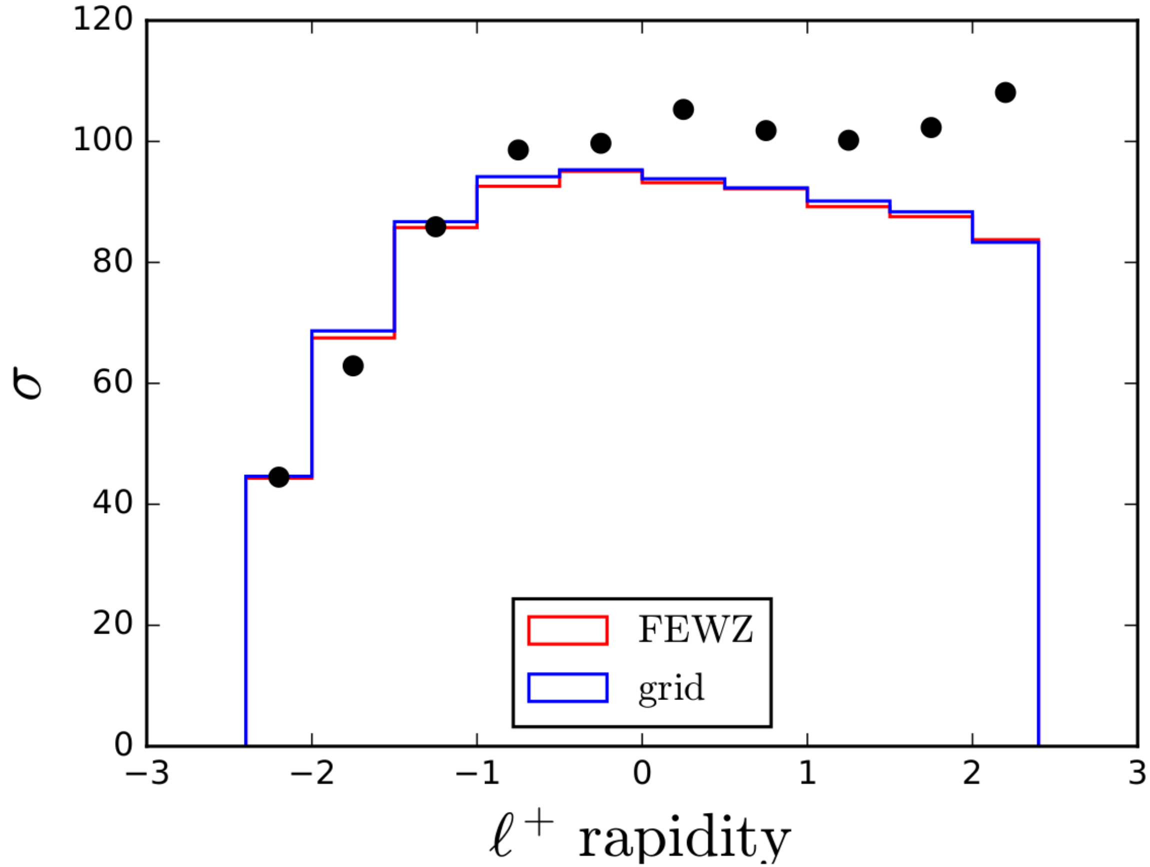
3. (3) Compare pp FEWZ to pp MCFM
4. (4) Generate APPLgrid grids
   - Using mcfm-bridge
   - Different Monte Carlo seeds
5. (5) Combine replica grids into a single PDF independent grid
   - Using applgrid-combine
6. (6) Convolute PDF independent grid with asymmetric PDFs to compare to pAFEWZ
7. (7) Add data and grid in nCTEQ++ to fit W/Z LHC data

Bringing it all together

Convoluted grids can then be compared to data and used in nCTEQ++ as theory predictions

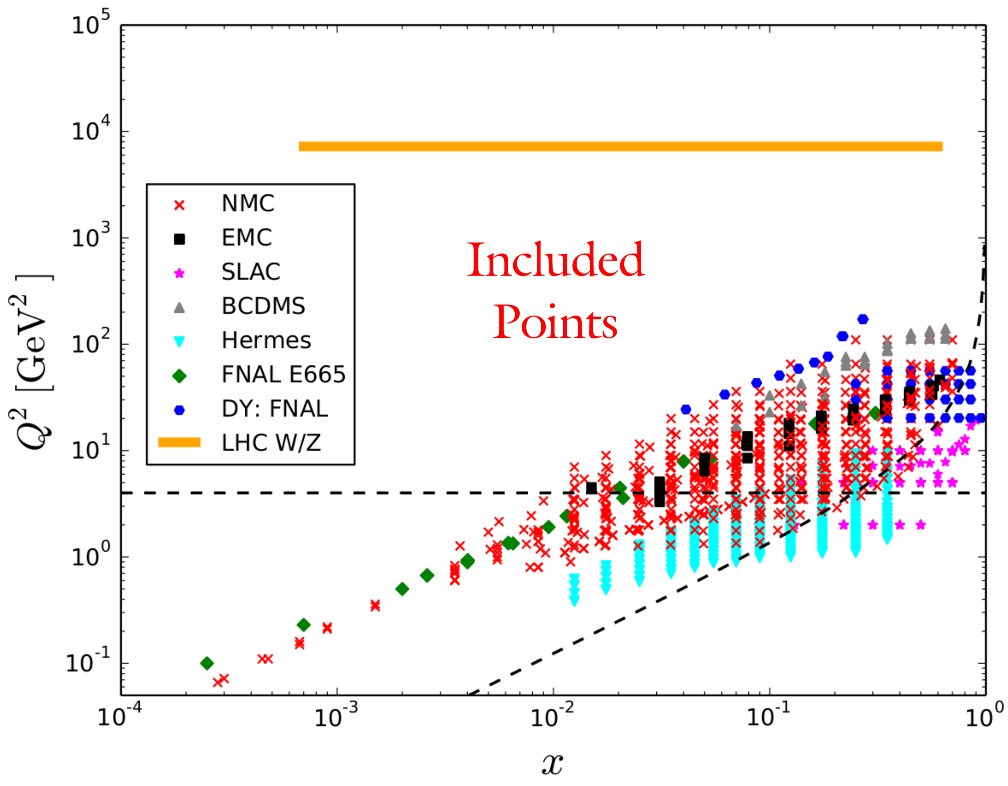


nCTEQ+LHC

pPb Data for nCTEQ+LHC

No LHC data in any previous nCTEQ fit

- New gridded theory predictions would make this possible



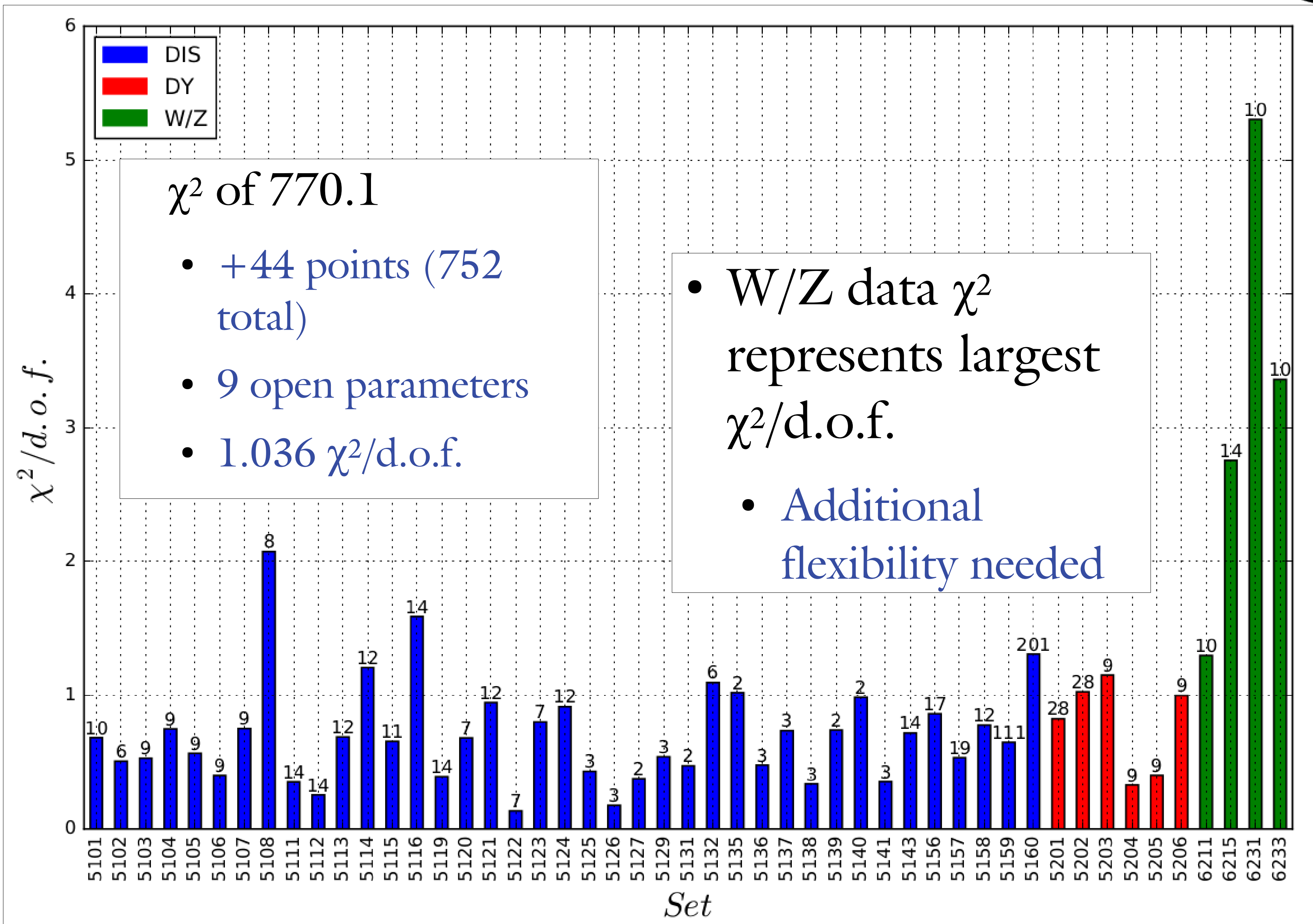
ATLAS:

- $d\sigma(W^- \to \ell^- \nu)/dy$
  ID: 6211 Npts: 10
- $d\sigma(Z \to \ell^+ \ell^-)/dy$
  ID: 6215 Npts: 14

CMS:

- $d\sigma(W^- \to \ell^- \nu)/dy$
  ID: 6231 Npts: 10
- $d\sigma(W^+ \to \ell^+ \nu)/dy$
  ID: 6233 Npts: 10

nCTEQ+LHC

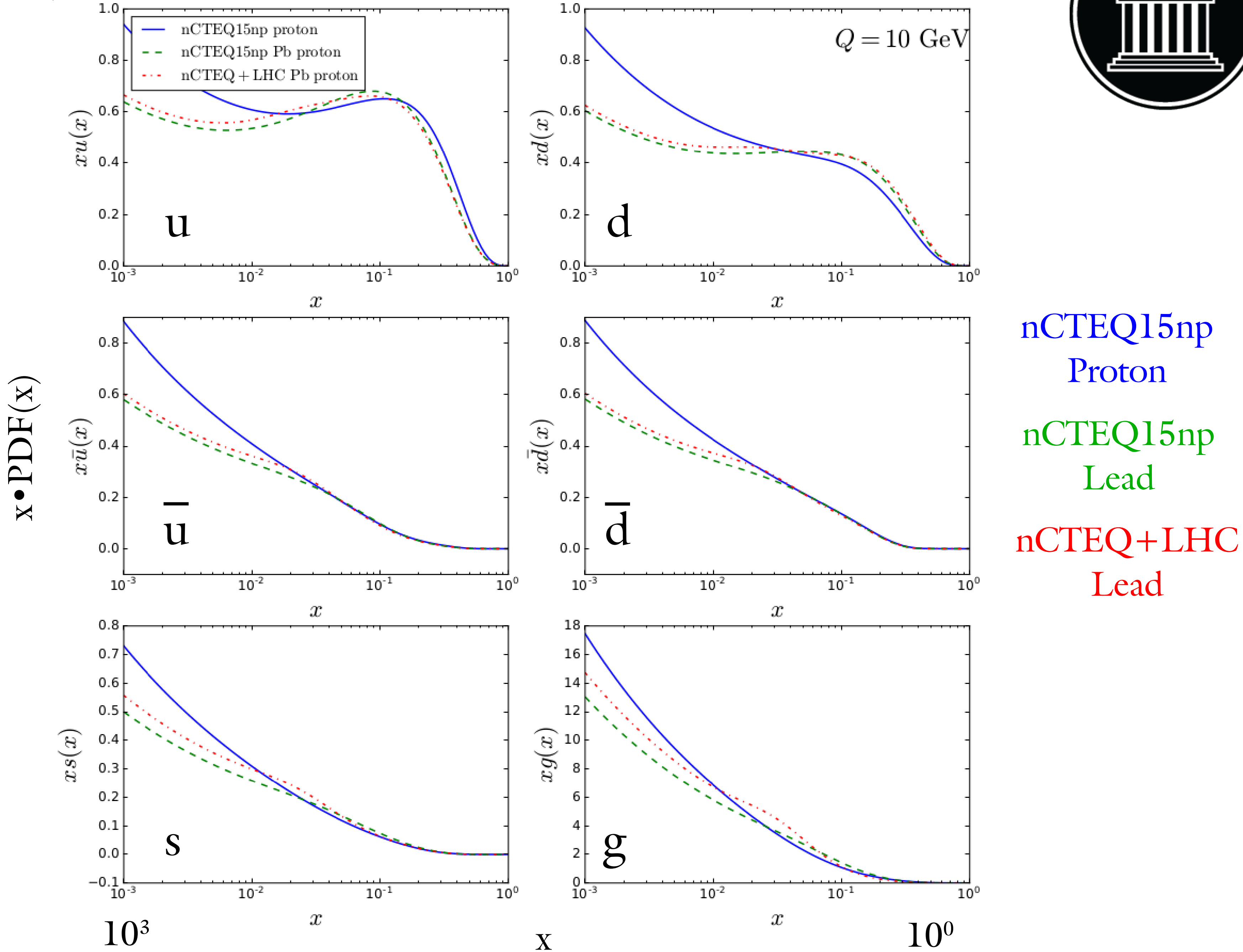


χ^2 of 770.1

- +44 points (752 total)
- 9 open parameters
- 1.036 χ^2/d.o.f.

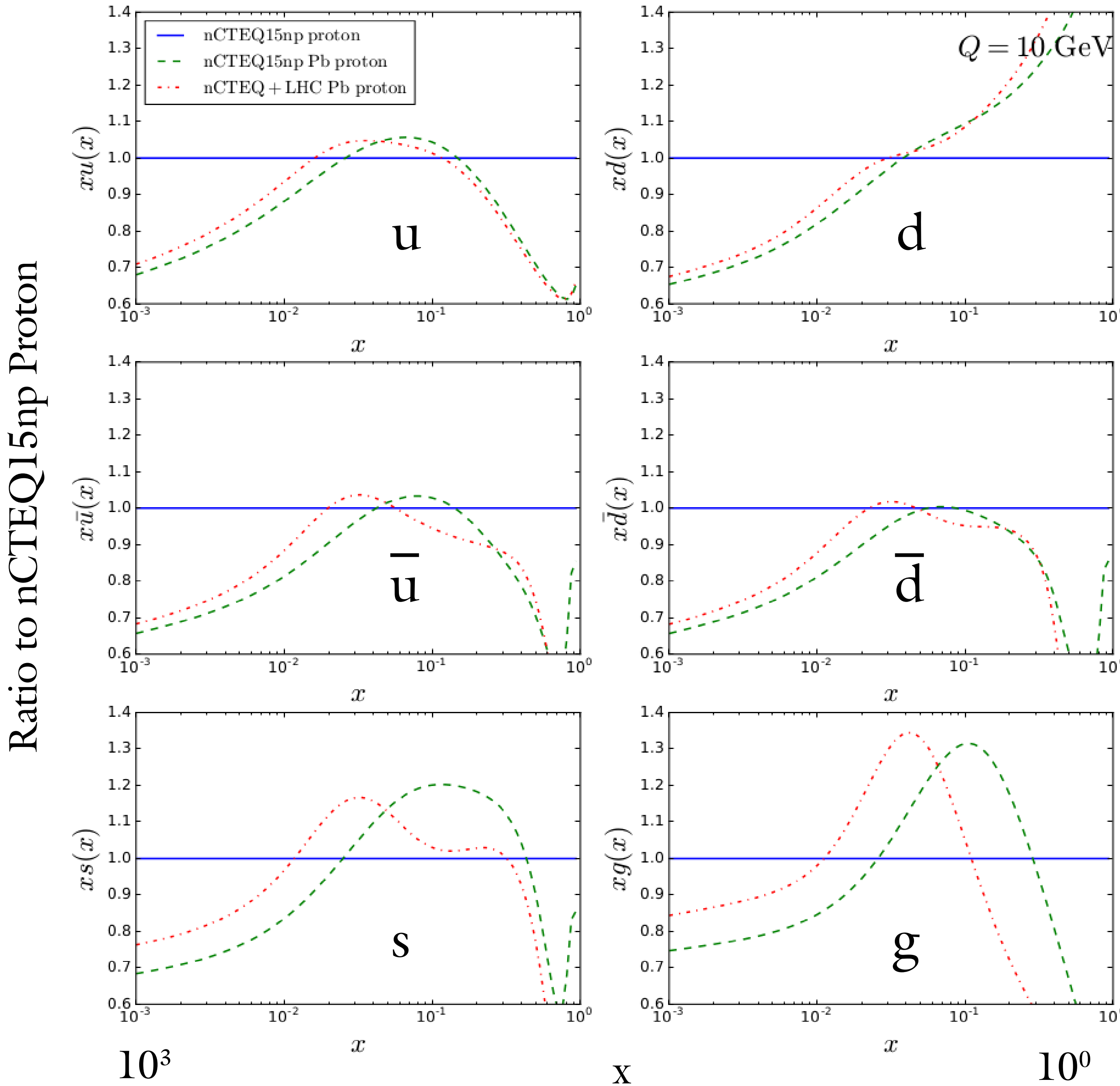
- W/Z data χ^2 represents largest χ^2/d.o.f.
  - Additional flexibility needed

nCTEQ+LHC



nCTEQ15np proton
nCTEQ15np Pb proton
nCTEQ + LHC Pb proton
Q = 10 GeV
u
d
ū
d̄
s
g
xu(x)
xd(x)
xū(x)
xd̄(x)
xs(x)
xg(x)
x
x•PDF(x)
10³
10⁰
nCTEQ15np Proton
nCTEQ15np Lead
nCTEQ+LHC Lead

nCTEQ+LHC

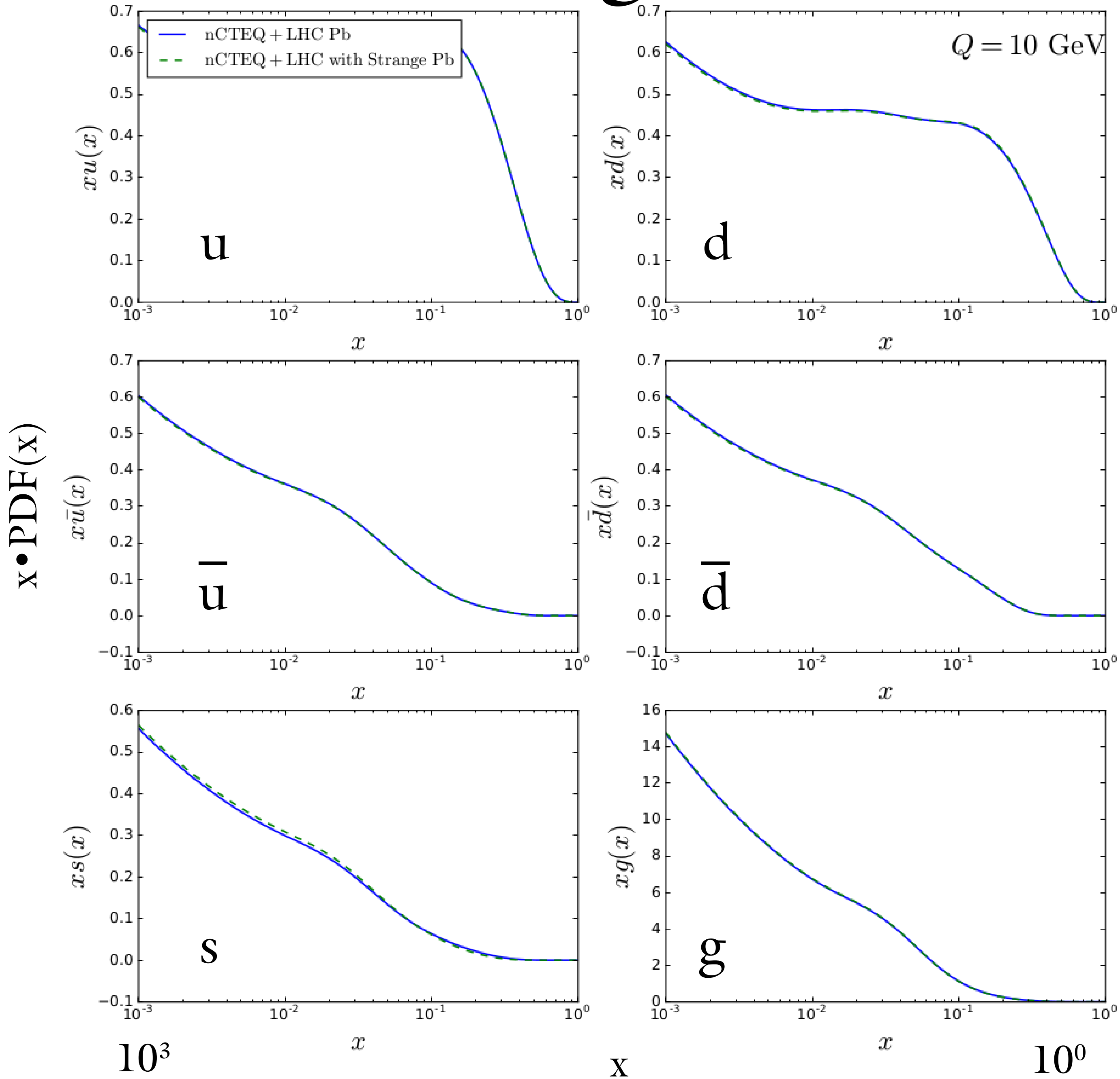


nCTEQ15np Proton

nCTEQ15np Lead

nCTEQ+LHC Lead

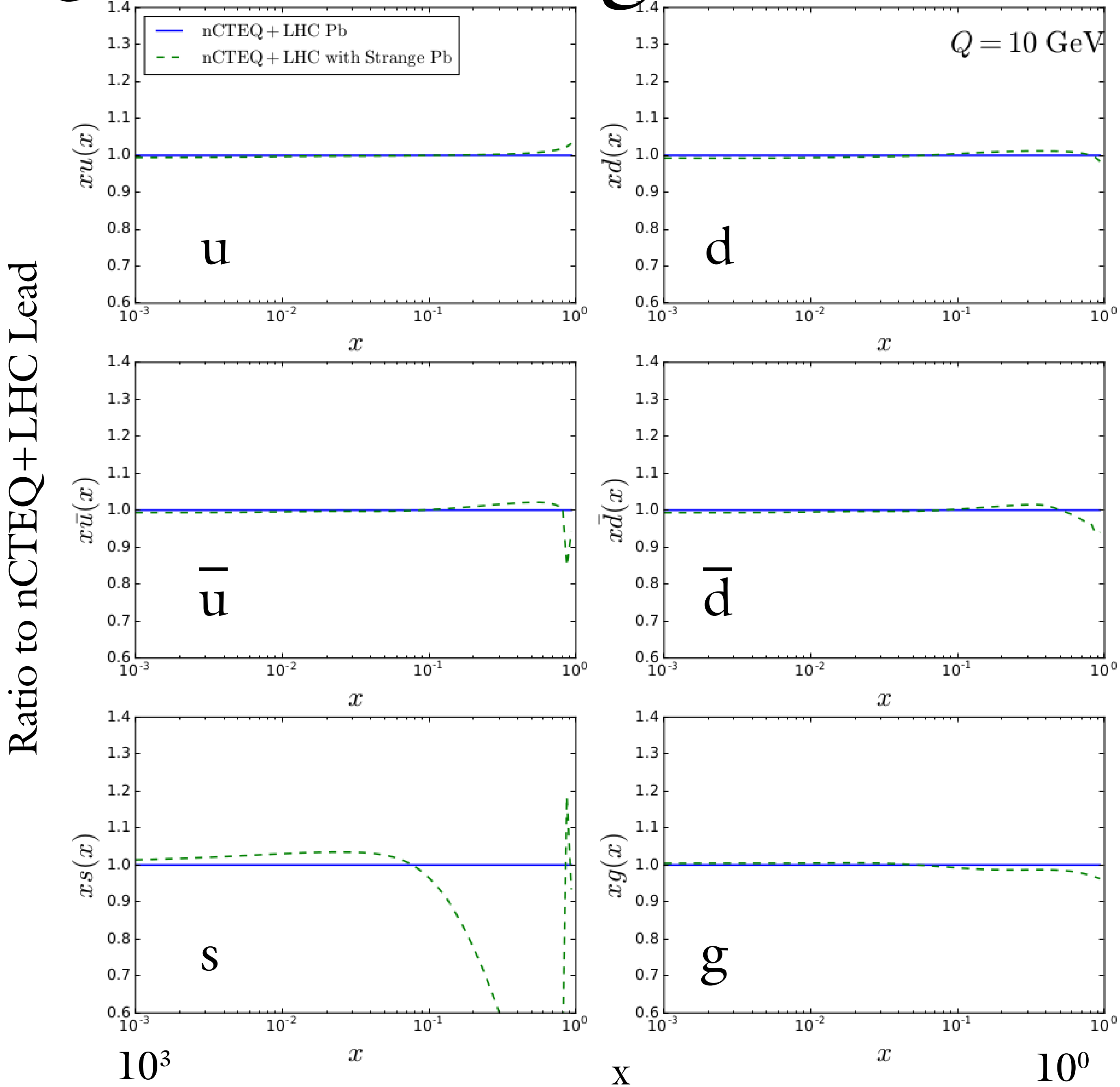
nCTEQ+LHC strange



nCTEQ+LHC Lead

nCTEQ+LHC Strange Lead

nCTEQ+LHC strange

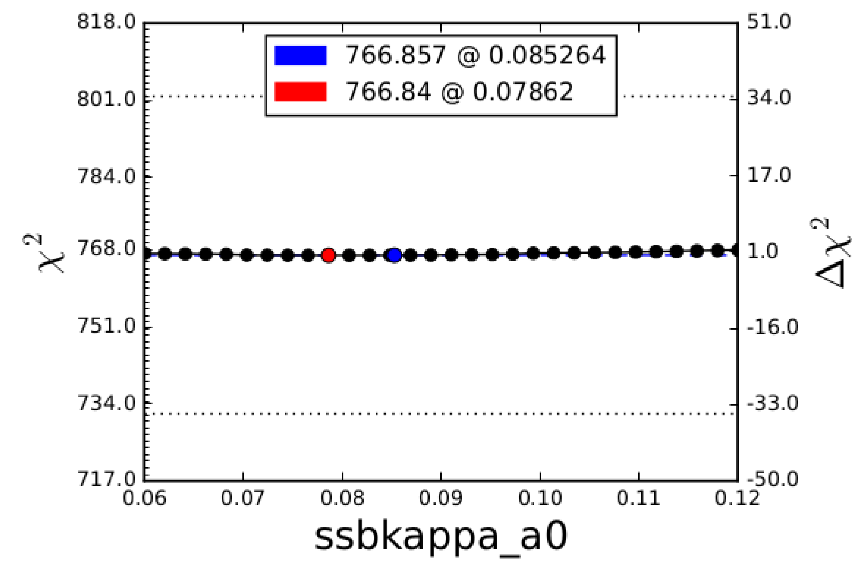


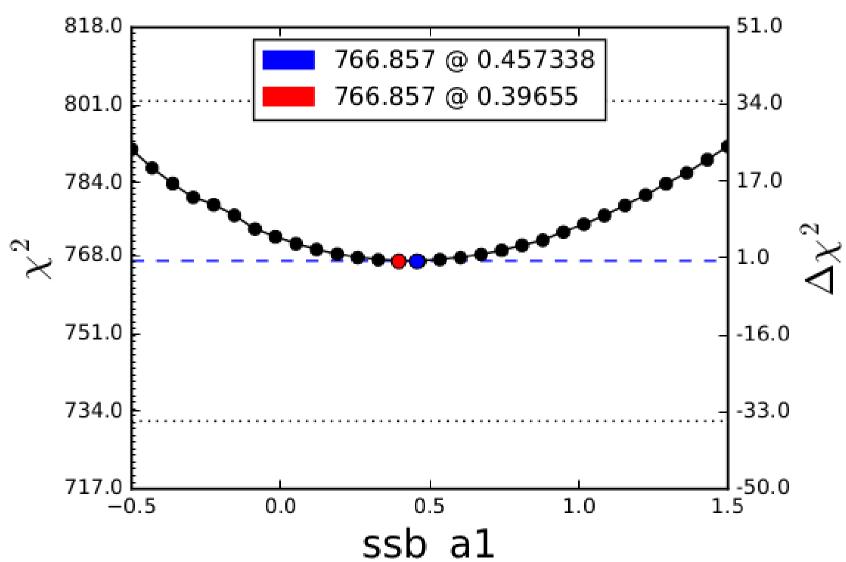
nCTEQ+LHC Lead

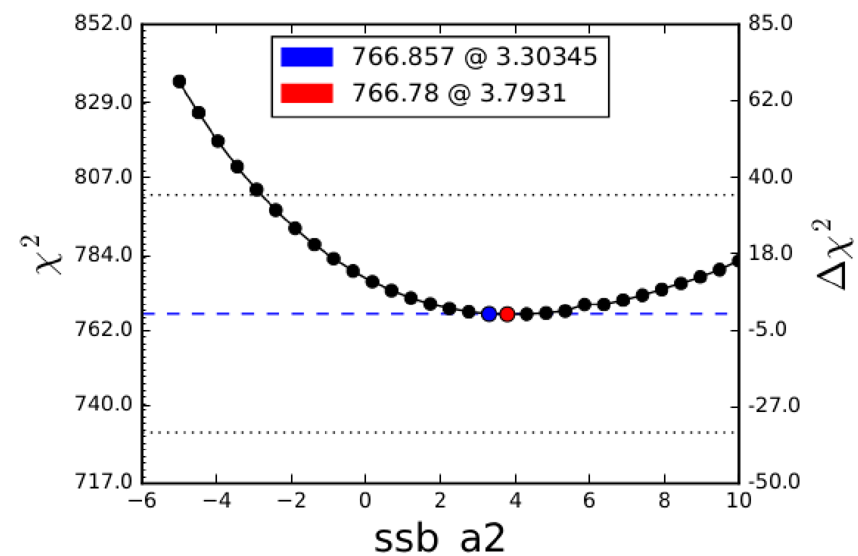
nCTEQ+LHC Strange Lead

nCTEQ+LHC strange

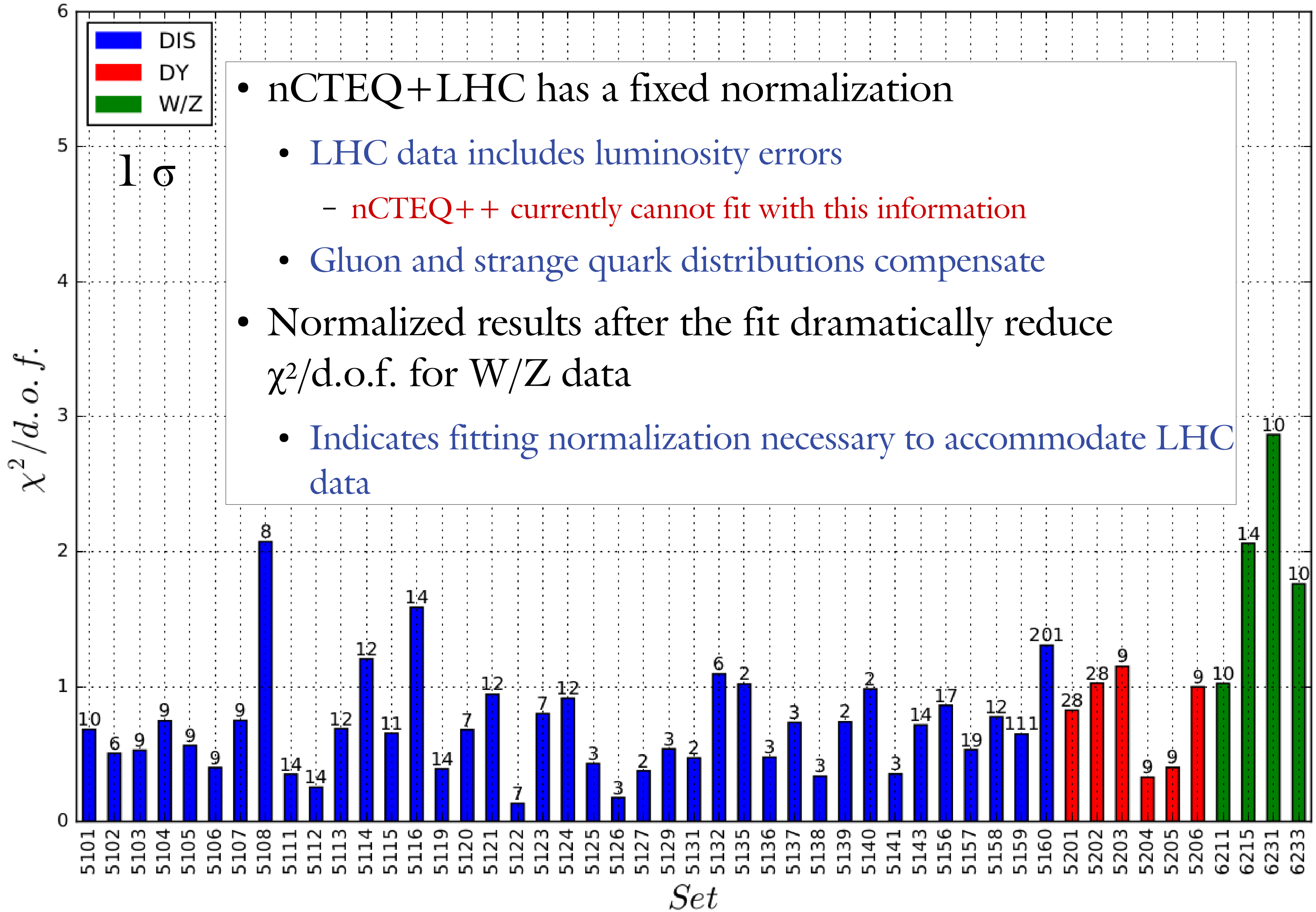
- Opening the strange parameter had little effect on fit
  - Possibly due to small number of data points
  - Strange asymmetry remained fixed in parameterization
  - κ=1/2 from proton fit
- Parameter scans show insensitivity in strange







Normalization



- nCTEQ+LHC has a fixed normalization
  - LHC data includes luminosity errors
    - nCTEQ++ currently cannot fit with this information
  - Gluon and strange quark distributions compensate
- Normalized results after the fit dramatically reduce χ^2/d.o.f. for W/Z data
  - Indicates fitting normalization necessary to accommodate LHC data

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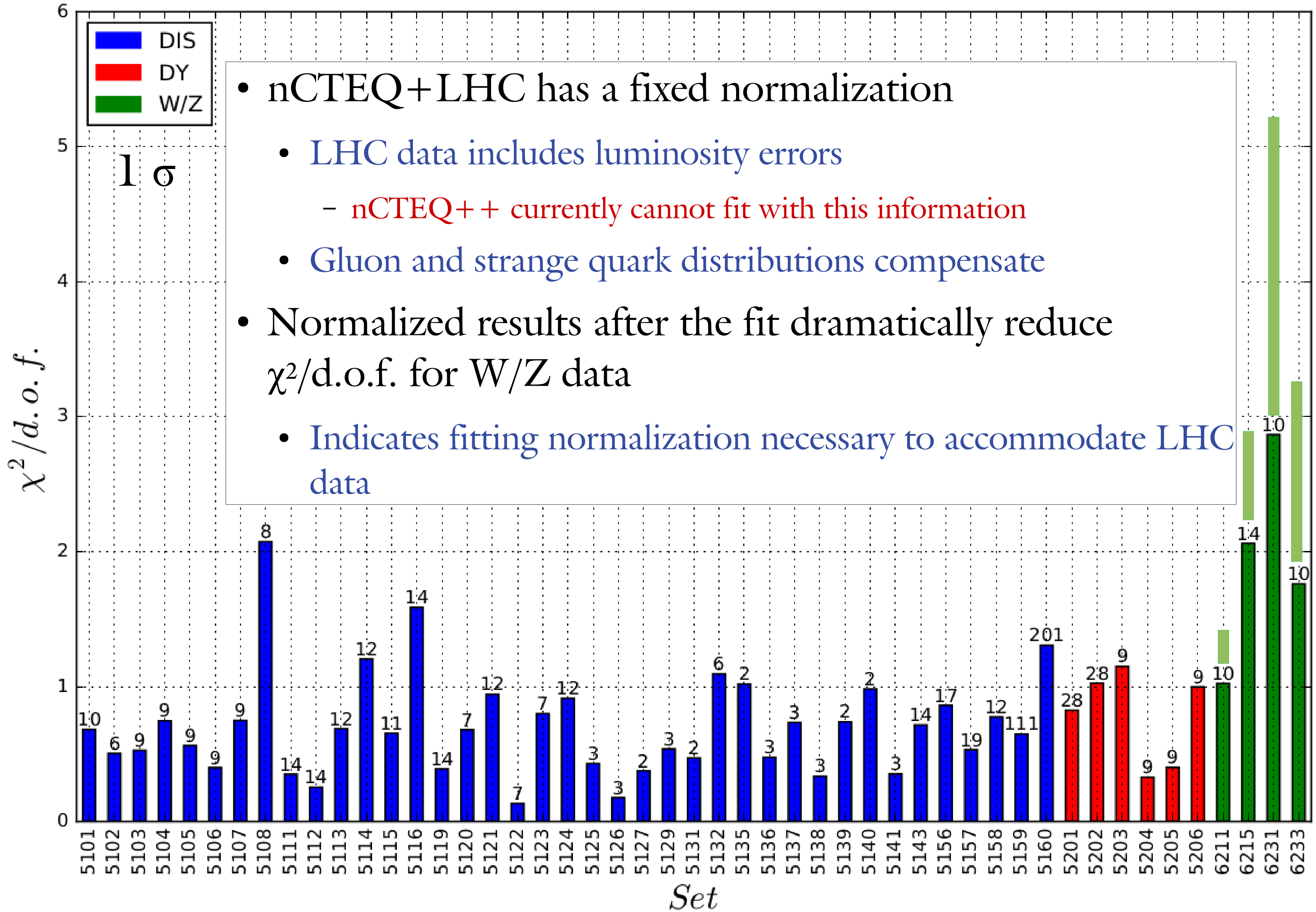
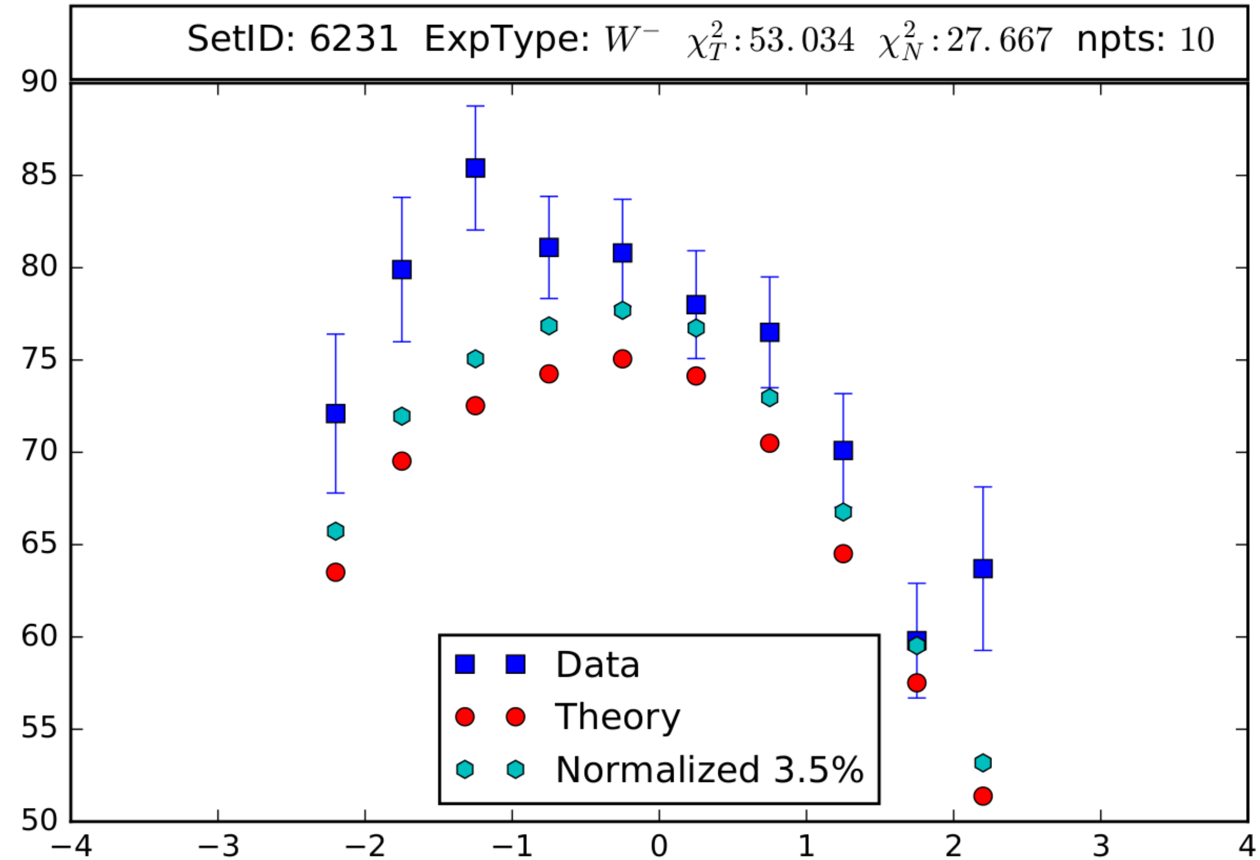


Normalization



1σ normalization applied to LHC sets

- Improved χ^2/d.o.f.
- Additional normalization could improve χ^2 more

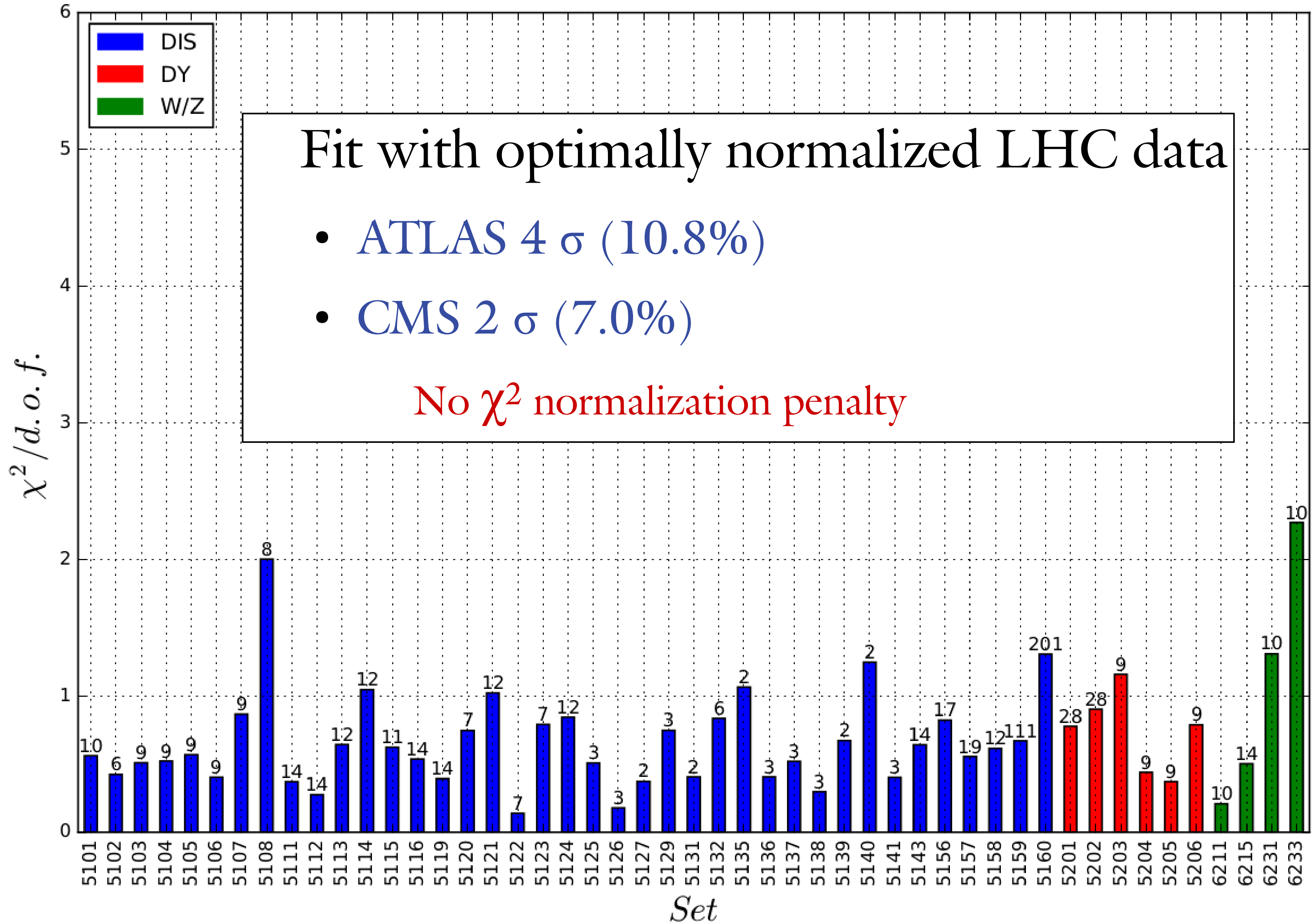
Penalty should be implemented to the χ^2

	Data ID:	6211	6231	6233	6215
`nCTEQ15-np`	χ^2per d.o.f:	1.55	6.91	7.73	3.16
Reweighting	χ^2per d.o.f:	0.87	3.27	2.95	1.76
`nCTEQ+LHC`	χ^2per d.o.f:	1.30	5.30	3.36	2.75
`nCTEQ+LHC` $(1 \times \sigma_N)$	χ^2per d.o.f:	0.92(+0.10)	2.77(+0.10)	1.66(+0.10)	1.96(+0.07)
`nCTEQ+LHC` $(4\sigma_N^{ATLAS},\ 2\sigma_N^{CMS})$	χ^2per d.o.f:	0.42(+1.60)	1.33(+0.40)	1.39(+0.40)	0.94(+1.14)

Normalized

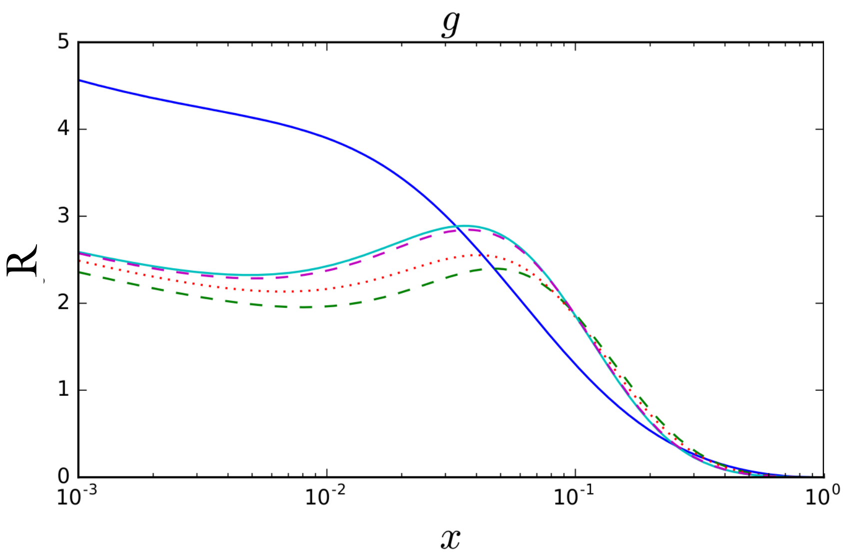
Penalty

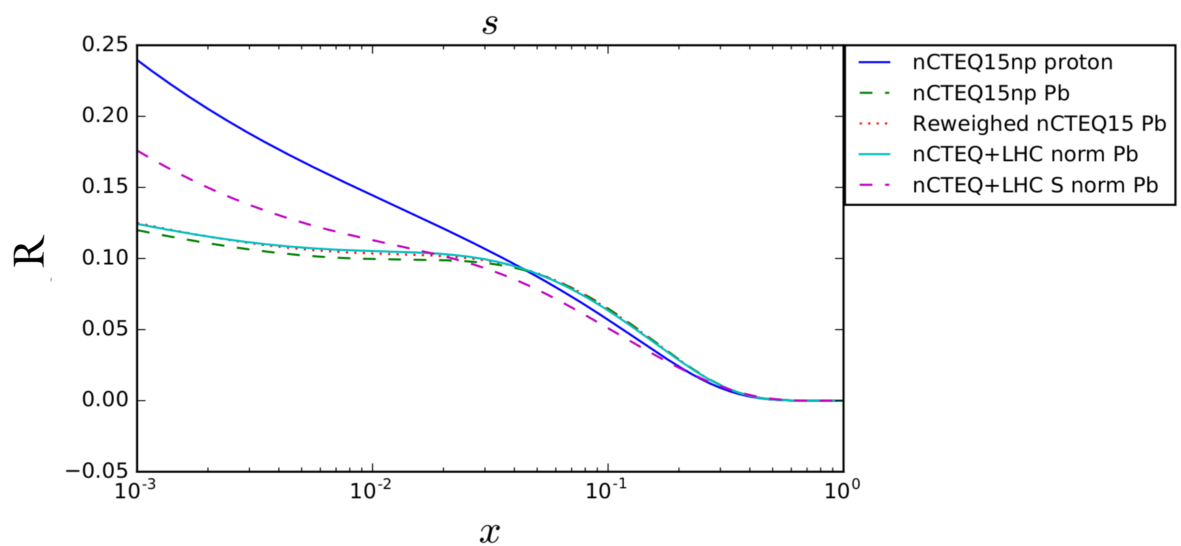
Fit with Normalization



Fit with Normalization

- Gluon and Strange quark able adjust to shape
  - Less of the fit is tied to compensating for normalization
- Validation of the results from reweighting
  - Reweighting indicated larger strange contribution at low x
  - Normalized fits show this to greater degree





Ratio of PDFs to nCTEQ15np lead

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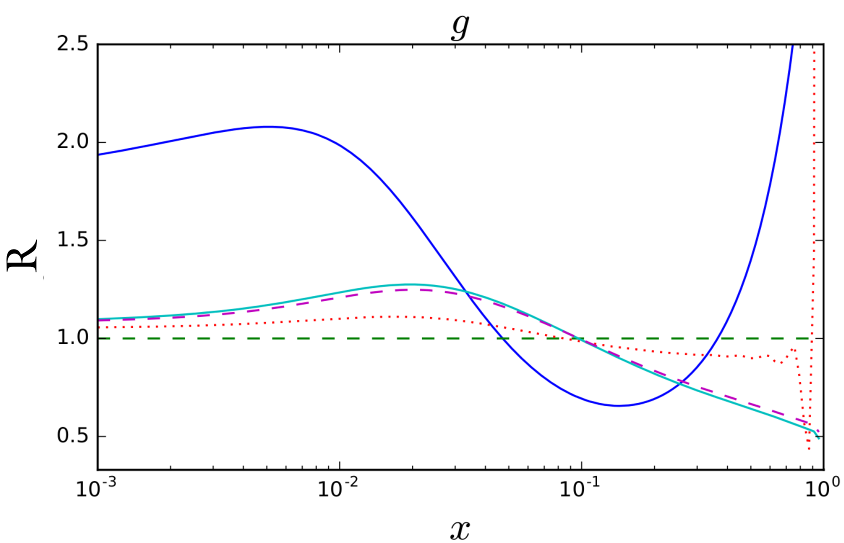
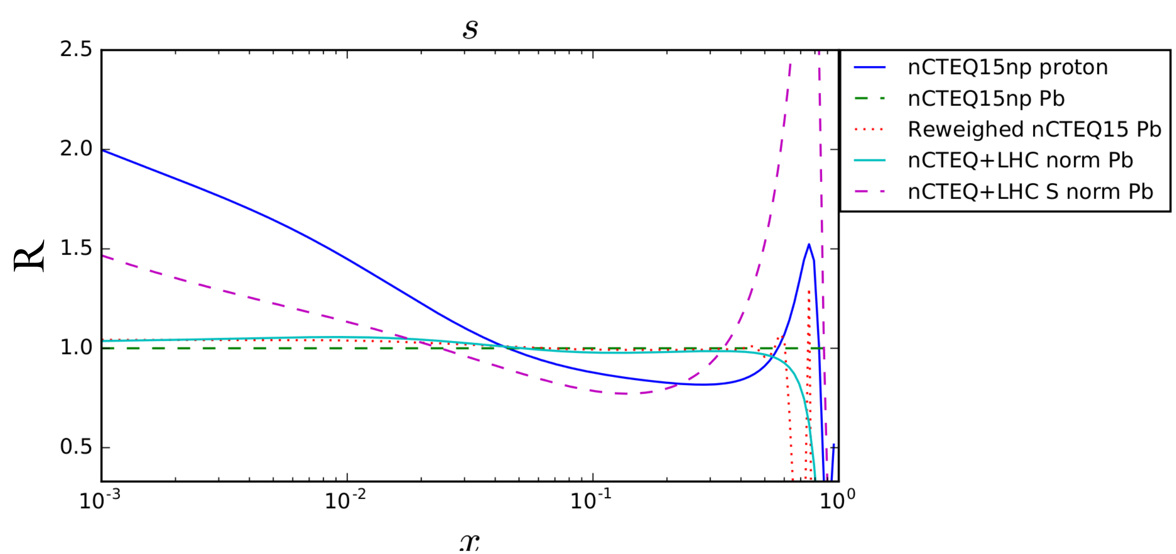




Ratio of PDFs to nCTEQ15np lead

Conclusions

Other Topics

ManeParse

- Mathematica Package
- Parses and interpolates PDFs
- Calculation and visualization functionalities

Heavy Flavor Variable Number Schemes

- Work done with xFitter collaboration
- Studies the effect of shifting heavy quark mass thresholds for PDF fitting

Conclusions

- Inclusion of W/Z LHC data improves nCTEQ15 fit
  - Better description of shape
    - Normalization important
      - This can be adjusted post-fit
  - Data coverage in new kinematic region eliminates extrapolation
    - Improves shape of fit in positive rapidity
  - Reweighting analysis did indicate direction of new fit
- nCTEQ parameterization remains overly restrictive
  - Fixed strange and anti-strange quark contributions
  - κ parameter from the proton fit
- Fitting normalization is necessary
  - Greatly reduces χ^2
  - Allows opening strange quark parameters

Next Steps nCTEQ+LHC

- More data available from the LHC
  - nCTEQ+LHC only contains a fraction of the data sets used in the reweighting study
    - e.g. LHCb, ALICE, ATLAS W^+, PbPb sets
  - MCFM provides expansive library of processes (~1000)
    - Gridded theory predictions for a variety of types of data
- Dynamic normalization fitting with χ^2 penalties
    - Module nearly complete
- Hessian Error analysis to describe error bars on PDFs
  - Delicate for nuclear fitting due to lack of data and flat parameter space
    - In progress

MCFM Processes Library (v6.8)

Citation

nproc	$f(p_1)+f(p_2)\to\dots$	Order
1	$W^+(\to\nu(p_3)+e^+(p_4))$	NLO
6	$W^-(\to e^-(p_3)+\bar\nu(p_4))$	NLO
11	$W^+(\to\nu(p_3)+e^+(p_4))+f(p_5)$	NLO
12	$W^+(\to\nu(p_3)+e^+(p_4))+\bar b(p_5)$	NLO
13	$W^+(\to\nu(p_3)+e^+(p_4))+\bar c(p_5)$	NLO
14	$W^+(\to\nu(p_3)+e^+(p_4))+\bar c(p_5)$[massless]	LO
16	$W^-(\to e^-(p_3)+\bar\nu(p_4))+f(p_5)$	NLO
17	$W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)$	NLO
18	$W^-(\to e^-(p_3)+\bar\nu(p_4))+c(p_5)$	NLO
19	$W^-(\to e^-(p_3)+\bar\nu(p_4))+c(p_5)$[massless]	LO

nproc	$f(p_1)+f(p_2)\to\dots$	Order
1	$W^+(\to\nu(p_3)+e^+(p_4))$	NLO
6	$W^-(\to e^-(p_3)+\bar\nu(p_4))$	NLO
11	$W^+(\to\nu(p_3)+e^+(p_4))+f(p_5)$	NLO
12	$W^+(\to\nu(p_3)+e^+(p_4))+\bar b(p_5)$	NLO
13	$W^+(\to\nu(p_3)+e^+(p_4))+\bar c(p_5)$	NLO
14	$W^+(\to\nu(p_3)+e^+(p_4))+\bar c(p_5)$[massless]	LO
16	$W^-(\to e^-(p_3)+\bar\nu(p_4))+f(p_5)$	NLO
17	$W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)$	NLO
18	$W^-(\to e^-(p_3)+\bar\nu(p_4))+c(p_5)$	NLO
19	$W^-(\to e^-(p_3)+\bar\nu(p_4))+c(p_5)$[massless]	LO
20	$W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+\bar b(p_6)$[massive]	NLO
21	$W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+\bar b(p_6)$	NLO
22	$W^+(\to\nu(p_3)+e^+(p_4))+f(p_5)+f(p_6)$	NLO
23	$W^+(\to\nu(p_3)+e^+(p_4))+f(p_5)+f(p_6)+f(p_7)$	LO
24	$W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+\bar b(p_6)+f(p_7)$	LO
25	$W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)+\bar b(p_6)$[massive]	NLO
26	$W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)+\bar b(p_6)$	NLO
27	$W^-(\to e^-(p_3)+\bar\nu(p_4))+f(p_5)+f(p_6)$	NLO
28	$W^-(\to e^-(p_3)+\bar\nu(p_4))+f(p_5)+f(p_6)+f(p_7)$	LO
29	$W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)+\bar b(p_6)+f(p_7)$	LO
31	$Z^0(\to e^-(p_3)+e^+(p_4))$	NLO
32	$Z^0(\to 3\times(\nu(p_3)+\bar\nu(p_4)))$	NLO
33	$Z^0(\to b(p_3)+\bar b(p_4))$	NLO
34	$Z^0(\to 3\times(d(p_3)+\bar d(p_4)))$	NLO
35	$Z^0(\to 2\times(u(p_3)+\bar u(p_4)))$	NLO
36	$Z\to t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to\bar b(p_6)+e^-(p_7)+\bar\nu(p_8))$	LO
41	$Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)$	NLO
42	$Z_0(\to 3\times(\nu(p_3)+\bar\nu(p_4)))+f(p_5)$	NLO
43	$Z^0(\to b(p_3)+\bar b(p_4))+f(p_5)$	NLO
44	$Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)+f(p_6)$	NLO
45	$Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)+f(p_6)+f(p_7)$	LO
46	$Z^0(\to 3\times(\nu(p_3)+\bar\nu(p_4)))+f(p_5)+f(p_6)$	NLO
47	$Z^0(\to 3\times(\nu(p_3)+\bar\nu(p_4)))+f(p_5)+f(p_6)+f(p_7)$	LO
50	$Z^0(\to e^-(p_3)+e^+(p_4))+b(p_5)+\bar b(p_6)$[massive]	LO
51	$Z^0(\to e^-(p_3)+e^+(p_4))+b(p_5)+\bar b(p_6)$	NLO
52	$Z_0(\to 3\times(\nu(p_3)+\bar\nu(p_4)))+b(p_5)+\bar b(p_6)$	NLO
53	$Z^0(\to b(p_3)+\bar b(p_4))+b(p_5)+\bar b(p_6)$	NLO
54	$Z^0(\to e^-(p_3)+e^+(p_4))+b(p_5)+\bar b(p_6)+f(p_7)$	LO

101	$Z^0(\to e^-(p_3)+e^+(p_4))+H(\to b(p_5)+\bar b(p_6))$	NLO
102	$Z^0(\to 3\times(\nu(p_3)+\bar\nu(p_4)))+H(\to b(p_5)+\bar b(p_6))$	NLO
103	$Z^0(\to b(p_3)+\bar b(p_4))+H(\to b(p_5)+\bar b(p_6))$	NLO
104	$Z^0(\to e^-(p_3)+e^+(p_4))+H(\to\gamma(p_5)+\gamma(p_6))$	NLO
105	$Z^0(\to 3\times(\nu(p_3)+\bar\nu(p_4)))+H(\to\gamma(p_5)+\gamma(p_6))$	NLO
106	$Z^0(\to e^-(p_3)+e^+(p_4))+H(\to W^+(\nu(p_5),e^+(p_6))W^-(e^-(p_7),\bar\nu(p_8)))$	NLO
107	$Z^0(\to 3\times(\nu(p_3)+\bar\nu(p_4)))+H(\to W^+(\nu(p_5),e^+(p_6))W^-(e^-(p_7),\bar\nu(p_8)))$	NLO
108	$Z^0(\to b(p_3)+\bar b(p_4))+H(\to W^+(\nu(p_5),e^+(p_6))W^-(e^-(p_7),\bar\nu(p_8)))$	NLO
109	$Z^0(\to e^-(p_3)+e^+(p_4))+H(\to Z(e^-(p_5),e^+(p_6))+Z(\mu^-(p_7),\mu^+(p_8)))$	NLO
111	$H(\to b(p_3)+\bar b(p_4))$	NLO
112	$H(\to\tau^-(p_3)+\tau^+(p_4))$	NLO
113	$H(\to W^+(\nu(p_3)+e^+(p_4))+W^-(e^-(p_5)+\bar\nu(p_6)))$	NLO
114	$H(\to W^+(\nu(p_3)+e^+(p_4))+W^-(q(p_5)+\bar q(p_6)))$	NLO
115	$H(\to W^+(\nu(p_3)+e^+(p_4))+W^-(q(p_5)+\bar q(p_6)))$[rad.in.dk]	NLO
116	$H(\to Z^0(e^-(p_3)+e^+(p_4))+Z^0(\mu^-(p_5)+\mu^+(p_6)))$	NLO
117	$H(\to Z^0(3\times(\nu(p_3)+\bar\nu(p_4)))+Z^0(\mu^-(p_5)+\mu^+(p_6)))$	NLO
118	$H(\to Z^0(\mu^-(p_3)+\mu^+(p_4))+Z^0(b(p_5)+\bar b(p_6)))$	NLO
119	$H(\to\gamma(p_3)+\gamma(p_4))$	NLO
120	$H(\to Z^0(\mu^-(p_3)+\mu^+(p_4))+\gamma(p_5))$	NLO
121	$H(\to Z^0(3\times(\nu(p_3)+\bar\nu(p_4)))+\gamma(p_5))$	NLO

56	$Z^0(\to e^-(p_3)+e^+(p_4))+c(p_5)+\bar c(p_6)$	NLO
61	$W^+(\to\nu(p_3)+e^+(p_4))+W^-(\to e^-(p_5)+\bar\nu(p_6))$	NLO
62	$W^+(\to\nu(p_3)+e^+(p_4))+W^-(\to q(p_5)+\bar q(p_6))$	NLO
63	$W^+(\to\nu(p_3)+e^+(p_4))+W^-(\to q(p_5)+\bar q(p_6))$[rad.in.dk]	NLO
64	$W^-(\to e^-(p_3)+\bar\nu(p_4))W^+(\to q(p_5)+\bar q(p_6))$	NLO
65	$W^-(\to e^-(p_3)+\bar\nu(p_4))W^+(\to q(p_5)+\bar q(p_6))$[rad.in.dk]	NLO
66	$W^+(\to\nu(p_3)+e^+(p_4))+W^-(\to e^-(p_5)+\bar\nu(p_6))+f(p_7)$	LO
69	$W^+(\to\nu(p_3)+e^+(p_4))+W^-(\to e^-(p_5)+\bar\nu(p_6))$[no pol]	LO
71	$W^+(\to\nu(p_3)+\mu^+(p_4))+Z^0(\to e^-(p_5)+e^+(p_6))$	NLO
72	$W^+(\to\nu(p_3)+\mu^+(p_4))+Z^0(\to 3\times(\nu_e(p_5)+\bar\nu_e(p_6)))$	NLO
73	$W^+(\to\nu(p_3)+\mu^+(p_4))+Z^0(\to b(p_5)+\bar b(p_6))$	NLO
74	$W^+(\to\nu(p_3)+\mu^+(p_4))+Z^0(\to 3\times(d(p_5)+\bar d(p_6)))$	NLO
75	$W^+(\to\nu(p_3)+\mu^+(p_4))+Z^0(\to 2\times(u(p_5)+\bar u(p_6)))$	NLO
76	$W^-(\to\mu^-(p_3)+\bar\nu(p_4))+Z^0(\to e^-(p_5)+e^+(p_6))$	NLO
77	$W^-(\to e^-(p_3)+\bar\nu(p_4))+Z^0(\to 3\times(\nu_e(p_5)+\bar\nu_e(p_6)))$	NLO
78	$W^-(\to e^-(p_3)+\bar\nu(p_4))+Z^0(\to b(p_5)+\bar b(p_6))$	NLO
79	$W^-(\to e^-(p_3)+\bar\nu(p_4))+Z^0(\to 3\times(d(p_5)+\bar d(p_6)))$	NLO
80	$W^-(\to e^-(p_3)+\bar\nu(p_4))+Z^0(\to 2\times(u(p_5)+\bar u(p_6)))$	NLO
81	$Z^0(\to e^-(p_3)+e^+(p_4))+Z^0(\to\mu^-(p_5)+\mu^+(p_6))$	NLO
82	$Z^0(\to e^-(p_3)+e^+(p_4))+Z^0(\to 3\times(\nu(p_5)+\bar\nu(p_6)))$	NLO
83	$Z^0(\to e^-(p_3)+e^+(p_4))+Z^0(\to b(p_5)+\bar b(p_6))$	NLO
84	$Z^0(\to b(p_3)+\bar b(p_4))+Z^0(\to 3\times(\nu(p_5)+\bar\nu(p_6)))$	NLO
85	$Z^0(\to e^-(p_3)+e^+(p_4))+Z^0(\to 3\times(\nu(p_5)+\bar\nu(p_6)))+f(p_7)$	LO
86	$Z^0(\to\mu^-(p_3)+\mu^+(p_4))+Z^0(\to e^-(p_5)+e^+(p_6))$[no gamma*]	NLO
87	$Z^0(\to e^-(p_3)+e^+(p_4))+Z^0(\to 3\times(\nu(p_5)+\bar\nu(p_6)))$[no gamma*]	NLO
88	$Z^0(\to e^-(p_3)+e^+(p_4))+Z^0(\to b(p_5)+\bar b(p_6))$[no gamma*]	NLO
89	$Z^0(\to b(p_3)+\bar b(p_4))+Z^0(\to 3\times(\nu(p_5)+\bar\nu(p_6)))$[no gamma*]	NLO
90	$Z^0(\to e^-(p_3)+e^+(p_4))+Z^0(\to e^-(p_5)+e^+(p_6))$	NLO
91	$W^+(\to\nu(p_3)+e^+(p_4))+H(\to b(p_5)+\bar b(p_6))$	NLO
92	$W^+(\to\nu(p_3)+e^+(p_4))+H(\to W^+(\nu(p_5),e^+(p_6))W^-(e^-(p_7),\bar\nu(p_8)))$	NLO
93	$W^+(\to\nu(p_3)+e^+(p_4))+H(\to Z(e^-(p_5),e^+(p_6))+Z(\mu^-(p_7),\mu^+(p_8)))$	NLO
94	$W^+(\to\nu(p_3)+e^+(p_4))+H(\to\gamma(p_5)+\gamma(p_6))$	NLO
96	$W^-(\to e^-(p_3)+\bar\nu(p_4))+H(\to b(p_5)+\bar b(p_6))$	NLO
97	$W^-(\to e^-(p_3)+\bar\nu(p_4))+H(\to W^+(\nu(p_5),e^+(p_6))W^-(e^-(p_7),\bar\nu(p_8)))$	NLO
98	$W^-(\to e^-(p_3)+\bar\nu(p_4))+H(\to Z(e^-(p_5),e^+(p_6))+Z(\mu^-(p_7),\mu^+(p_8)))$	NLO
99	$W^-(\to e^-(p_3)+\bar\nu(p_4))+H(\to\gamma(p_5)+\gamma(p_6))$	NLO

123	$H(\to W^+(\nu(p_3)+e^+(p_4))+W^-(e^-(p_5)+\bar\nu(p_6)))$ [top, bottom loops, exact]	LO
124	$H(\to W^+(\nu(p_3)+e^+(p_4))+W^-(e^-(p_5)+\bar\nu(p_6)))$ [only H, gg→WW intf.	LO
125	$H(\to W^+(\nu(p_3)+e^+(p_4))+W^-(e^-(p_5)+\bar\nu(p_6)))$ [$\|H\|^2$ and H,gg→WW intf.]	LO
126	$W^+(\nu(p_3)+e^+(p_4))+W^-(e^-(p_5)+\bar\nu(p_6))$ [gg only, (H + gg→WW) squared]	LO
127	$W^+(\nu(p_3)+e^+(p_4))+W^-(e^-(p_5)+\bar\nu(p_6))$ [(gg→WW) squared]	LO
128	$H(\to Z^0(e^-(p3)+e^+(p4))+Z^0(\mu^-(p5)+\mu^+(p6))$ [top, bottom loops, exact]	LO
129	$H(\to Z^0(e^-(p3)+e^+(p4))+Z^0(\mu^-(p5)+\mu^+(p6))$ [only H, gg→ZZ intf.]	LO
130	$H(\to Z^0(e^-(p3)+e^+(p4))+Z^0(\mu^-(p5)+\mu^+(p6))$ [$\|H\|^2$ and H,gg→ZZ intf.]	LO
131	$Z^0(e^-(p3)+e^+(p4))+Z^0(\mu^-(p5)+\mu^+(p6))$ [gg only, (H + gg→ZZ) squared]	LO
132	$Z^0(e^-(p3)+e^+(p4))+Z^0(\mu^-(p5)+\mu^+(p6))$ [(gg→ZZ) squared]	LO
1281	$H(\to e^-(p3)+e^+(p4)\nu_e(p5)+\bar\nu_e(p6)$ [top, bottom loops, exact]	LO
1311	$e^-(p3)+e^+(p4)+\nu_e(p5)+\bar\nu_e(p6)$ [gg only, (H + gg→ZZ) squared]	LO
1321	$e^-(p3)+e^+(p4)+\nu_e(p5)+\bar\nu_e(p6)$ [(gg→ZZ) squared]	LO
1282	$H(\to e^-(p3)+e^+(p4)+\nu(p5)+\bar\nu(p6)$ [top, bottom loops, exact]	LO
1312	$e^-(p3)+e^+(p4)+\nu(p5)+\bar\nu(p6)$ [gg only, (H + gg→ZZ) squared]	LO
1322	$e^-(p3)+e^+(p4)+\nu(p5)+\bar\nu(p6)$ [(gg→ZZ) squared]	LO
133	$H(\to Z^0(e^-(p3)+e^+(p4))+Z^0(\mu^-(p5)+\mu^+(p6)+f(p7))$ [intf,no p_7 cut]	LO
136	$H(\to b(p_3)+\bar b(p_4))+b(p_5)(+g(p_6))$	NLO
137	$H(\to b(p_3)+\bar b(p_4))+\bar b(p_5)(+b(p_6))$	(REAL)
138	$H(\to b(p_3)+\bar b(p_4))+b(p_5)+\bar b(p_6)$[both observed]	(REAL)

141	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))$	NLO
142	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))$ [rad.in.dk]	NLO
143	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))+f(p_9)$	LO
144	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))$ (uncorr)	NLO
145	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))$ [rad.in.dk],uncorr	NLO
146	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+q(p_7)+\bar q(p_8))$	NLO
147	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+q(p_7)+\bar q(p_8))$ [rad.in.top.dk]	NLO
148	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+q(p_7)+\bar q(p_8))$ [rad.in.W.dk]	NLO
149	$t(\to q(p_3)+\bar q(p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))$	NLO
150	$t(\to q(p_3)+\bar q(p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))$ [rad.in.top.dk]	NLO
151	$t(\to q(p_3)+\bar q(p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))$ [rad.in.W.dk]	NLO
157	$t\bar t$[for total Xsect]	NLO
158	$b\bar b$[for total Xsect]	NLO
159	$c\bar c$[for total Xsect]	NLO
160	$t\bar t+g$[for total Xsect]	LO
161	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+q(p_6)$[t-channel]	NLO
162	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+q(p_6)$[decay]	NLO
163	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+q(p_6)$[t-channel]$mb>0$	NLO
166	$\bar t(\to e^-(p_3)+\bar\nu(p_4)+\bar b(p_5))+q(p_6)$[t-channel]	NLO
167	$\bar t(\to e^-(p_3)+\bar\nu(p_4)+\bar b(p_5))+q(p_6)$[rad.in.dk]	NLO
168	$\bar t(\to e^-(p_3)+\bar\nu(p_4)+\bar b(p_5))+q(p_6)$[t-channel]$mb>0$	NLO
171	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar b(p_6))$[s-channel]	NLO
172	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar b(p_6))$[decay]	NLO
176	$\bar t(\to e^-(p_3)+\bar\nu(p_4)+\bar b(p_5))+b(p_6))$[s-channel]	NLO
177	$\bar t(\to e^-(p_3)+\bar\nu(p_4)+\bar b(p_5))+b(p_6))$[rad.in.dk]	NLO
180	$W^-(\to e^-(p_3)+\bar\nu(p_4))+t(p_5)$	NLO
181	$W^-(\to e^-(p_3)+\bar\nu(p_4))+t(\nu(p_5)+e^+(p_6)+b(p_7))$	NLO
182	$W^-(\to e^-(p_3)+\bar\nu(p_4))+t(\nu(p_5)+e^+(p_6)+b(p_7))$[rad.in.dk]	NLO
183	$W^-(\to e^-(p_3)+\bar\nu(p_4))+t(\nu(p_5)+e^+(p_6)+b(p_7))+b(p_8)$	LO
184	$W^-(\to e^-(p_3)+\bar\nu(p_4))+t(p_5)+b(p_6)$[massive b]	LO
185	$W^+(\to\nu(p_3)+e^+(p_4))+\bar t(p_5)$	NLO
186	$W^+(\to\nu(p_3)+e^+(p_4))+\bar t(e^-(p_5)+\bar\nu(p_6)+\bar b(p_7)$	NLO
187	$W^+(\to\nu(p_3)+e^+(p_4))+\bar t(e^-(p_5)+\bar\nu(p_6)+\bar b(p_7)$[rad.in.dk]	NLO

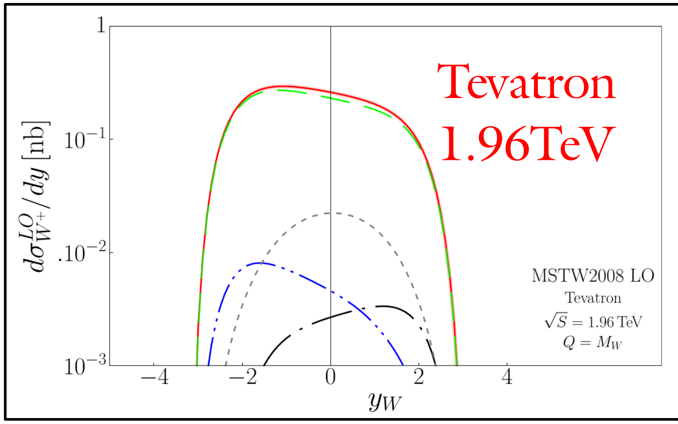
270	$H(\gamma(p_3)+\gamma(p_4))+f(p_5)+f(p_6)$[in heavy top limit]	NLO
271	$H(b(p_3)+\bar b(p_4))+f(p_5)+f(p_6)$[in heavy top limit]	NLO
272	$H(\tau^-(p_3)+\tau^+(p_4))+f(p_5)+f(p_6)$[in heavy top limit]	NLO
273	$H(\to W^+(\nu(p_3),e^+(p_4))W^-(e^-(p_5),\bar\nu(p_6)))+f(p_7)+f(p_8)$	NLO
274	$H(\to Z(e^-(p_3),e^+(p_4))Z(\mu^-(p_5),\mu^+(p_6)))+f(p_7)+f(p_8)$	NLO
275	$H(b(p_3)+\bar b(p_4))+f(p_5)+f(p_6)+f(p_7)$[in heavy top limit]	LO
276	$H(\tau^-(p_3)+\tau^+(p_4))+f(p_5)+f(p_6)+f(p_7)$[in heavy top limit]	LO
278	$H(\to W^+(\nu(p_3),e^+(p_4))W^-(e^-(p_5),\bar\nu(p_6)))+f(p_7)+f(p_8)+f(p_9)$	LO
279	$H(\to Z(e^-(p_3),e^+(p_4))Z(\mu^-(p_5),\mu^+(p_6)))+f(p_7)+f(p_8)+f(p_9)$	LO
280	$\gamma(p_3)+f(p_4)$	NLO+F
282	$f(p_1)+f(p_2)\to\gamma(p_3)+f(p_4)+f(p_5)$	LO
283	$f(p_1)+f(p_2)\to\gamma(p_3)+b(p_4)$	LO
284	$f(p_1)+f(p_2)\to\gamma(p_3)+c(p_4)$	LO
285	$f(p_1)+f(p_2)\to\gamma(p_3)+\gamma(p_4)$	NLO+F
286	$f(p_1)+f(p_2)\to\gamma(p_3)+\gamma(p_4)+f(p_5)$	NLO+F
287	$f(p_1)+f(p_2)\to\gamma(p_3)+\gamma(p_4)+\gamma(p_5)$	NLO+F
290	$W^+(\to\nu(p_3)+e^+(p_4))+\gamma(p_5)$	NLO+F
292	$W^+(\to\nu(p_3)+e^+(p_4))+\gamma(p_5)+f(p_6)$	LO
295	$W^-(\to e^-(p_3)+\bar\nu(p_4))+\gamma(p_5)$	NLO+F
297	$W^-(\to e^-(p_3)+\bar\nu(p_4))+\gamma(p_5)+f(p_6)$	LO
300	$Z^0(\to e^-(p_3)+e^+(p_4))+\gamma(p_5)$	NLO+F
301	$Z^0(\to e^-(p_3)+e^+(p_4))+\gamma(p_5)+\gamma(p_6)$	NLO +F
302	$Z^0(\to e^-(p_3)+e^+(p_4))+\gamma(p_5)+f(p_6)$	NLO + F
303	$Z^0(\to e^-(p_3)+e^+(p_4))+\gamma(p_5)+\gamma(p_6)+f(p_7)$	LO
304	$Z^0(\to e^-(p_3)+e^+(p_4))+\gamma(p_5)+f(p_6)+f(p_7)$	LO
305	$Z^0(\to 3(\nu(p_3)+\bar\nu(p_4)))+\gamma(p_5)$	NLO + F
306	$Z^0(\to 3(\nu(p_3)+\bar\nu(p_4)))+\gamma(p_5)+\gamma(p_6)$	NLO + F
307	$Z^0(\to 3(\nu(p_3)+\bar\nu(p_4)))+\gamma(p_5)+f(p_6)$	NLO + F
308	$Z^0(\to 3(\nu(p_3)+\bar\nu(p_4)))+\gamma(p_5)+\gamma(p_6)+f(p_7)$	LO
309	$Z^0(\to 3(\nu(p_3)+\bar\nu(p_4)))+\gamma(p_5)+f(p_6)+f(p_7)$	LO
311	$f(p_1)+b(p_2)\to W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+f(p_6)$	LO
316	$f(p_1)+b(p_2)\to W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)+f(p_6)$	LO
321	$f(p_1)+c(p_2)\to W^+(\to\nu(p_3)+e^+(p_4))+c(p_5)+f(p_6)$	LO
326	$f(p_1)+c(p_2)\to W^-(\to e^-(p_3)+\bar\nu(p_4))+c(p_5)+f(p_6)$	LO
331	$W^+(\to\nu(p_3)+e^+(p_4))+c(p_5)+f(p_6)$[c-s interaction]	LO
336	$W^-(\to e^-(p_3)+\bar\nu(p_4))+c(p_5)+f(p_6)$[c-s interaction]	LO
341	$f(p_1)+b(p_2)\to Z^0(\to e^-(p_3)+e^+(p_4))+b(p_5)+f(p_6)[+f(p_7)]$	NLO
342	$f(p_1)+b(p_2)\to Z^0(\to e^-(p_3)+e^+(p_4))+b(p_5)+f(p_6)[+\bar b(p_7)]$	(REAL)
346	$f(p_1)+b(p_2)\to Z^0(\to e^-(p_3)+e^+(p_4))+b(p_5)+f(p_6)+f(p_7)$	LO
347	$f(p_1)+b(p_2)\to Z^0(\to e^-(p_3)$ 84$e^+(p_4))+b(p_5)+f(p_6)+\bar b(p_7)$	LO

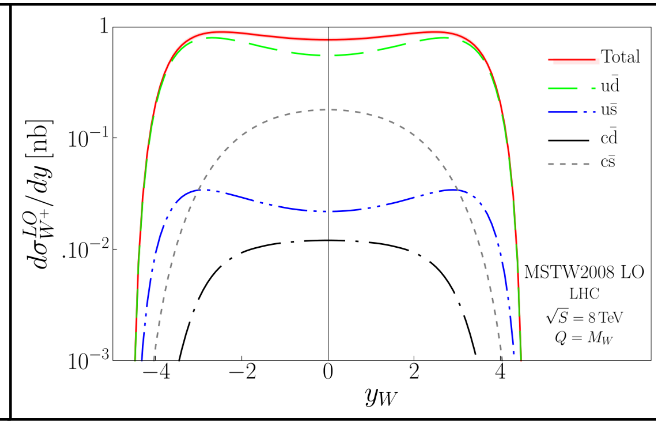
351	$f(p_1)+c(p_2)\to Z^0(\to e^-(p_3)+e^+(p_4))+c(p_5)+f(p_6)[+f(p_7)]$	NLO
352	$f(p_1)+c(p_2)\to Z^0(\to e^-(p_3)+e^+(p_4))+c(p_5)+f(p_6)[+\bar c(p_7)]$	(REAL)
356	$f(p_1)+c(p_2)\to Z^0(\to e^-(p_3)+e^+(p_4))+c(p_5)+f(p_6)+f(p_7)$	LO
357	$f(p_1)+c(p_2)\to Z^0(\to e^-(p_3)+e^+(p_4))+c(p_5)+f(p_6)+\bar c(p_7)$	LO
361	$c(p_1)+\bar s(p_2)\to W^+(\to\nu(p_3)+e^+(p_4))$[mc=0 in NLO]	NLO
362	$c(p_1)+\bar s(p_2)\to W^+(\to\nu(p_3)+e^+(p_4))$[massless corrections only]	NLO
363	$c(p_1)+\bar s(p_2)\to W^+(\to\nu(p_3)+e^+(p_4))$[massive charm in real]	NLO
370	$W^+(\to\nu(p_3)+e^+(p_4))+\gamma(p_5)+\gamma(p_6)$	LO
371	$W^-(\to e^-(p_3)+\bar\nu(p_4))+\gamma(p_5)+\gamma(p_6)$	LO
401	$W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)$ [1,2 or 3 jets, 4FNS]	NLO
402	$W^+(\to\nu(p_3)+e^+(p_4))+(b+\bar b)(p_5)$ [1 or 2 jets, 4FNS]	NLO
403	$W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+\bar b(p_6)$ [2 or 3 jets, 4FNS]	NLO
406	$W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)$ [1,2 or 3 jets, 4FNS]	NLO
407	$W^-(\to e^-(p_3)+\bar\nu(p_4))+(b+\bar b)(p_5)$ [1 or 2 jets, 4FNS]	NLO
408	$W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)+\bar b(p_6)$ [2 or 3 jets, 4FNS]	NLO
411	$f(p_1)+b(p_2)\to W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+f(p_6)$ [5FNS]	NLO
416	$f(p_1)+b(p_2)\to W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)+f(p_6)$ [5FNS]	NLO
421	$W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)$ [1,2 or 3 jets, 4FNS+5FNS]	NLO
426	$W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)$ [1,2 or 3 jets, 4FNS+5FNS]	NLO
431	$W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+\bar b(p_6)+f(p_7)$ [massive]	LO
436	$W^-(\to e^-(p_3)+\bar\nu(p_4))+b(p_5)+\bar b(p_6)+f(p_7)$ [massive]	LO
500	$W^+(\to\nu(p_3)+e^+(p_4))+t(p_5)+\bar t(p_6)$[massive]	NLO
501	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))+W^+(\nu(p_9),\mu^+(p_{10}))$	NLO
502	(same as process 501 but with radiation in decay)	NLO
503	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+q(p_7)+q\ (p_8))+W^+(\nu(p_9),\mu^+(p_{10}))$	NLO
506	$t(\to q(p_3)+q\ (p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))+W^+(\nu(p_9),\mu^+(p_{10}))$	NLO
510	$W^-(\to e^-(p_3)+\bar\nu(p_4))+t(p_5)+\bar t(p_6)$[massive]	NLO
511	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))+W^-(\mu^-(p_9),\bar\nu(p_{10}))$	NLO
512	(same as process 511 but with radiation in decay)	NLO
513	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to b\ (p_6)+q(p_7)+q\ (p_8))+W^-(\mu^-(p_9),\bar\nu(p_{10}))$	NLO
516	$t(\to q(p_3)+q\ (p_4)+b(p_5))+\bar t(\to b\ (p_6)+e^-(p_7)+\bar\nu(p_8))+W^-(\mu^-(p_9),\bar\nu(p_{10}))$	NLO
529	$Z^0(\to e^-(p_3)+e^+(p_4))+t(p5)+\bar t(p_6)$	LO
530	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to e^-(p_7)+\bar\nu(p_8)+b\ (p_6))+Z(e^-(p_9),e^+(p_{10}))$	LO
531	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to e^-(p_7)+\bar\nu(p_8)+b\ (p_6))+Z(b(p_9),b\ (p_{10}))$	LO
532	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to q(p_7)+\bar q(p_8)+b\ (p_6))+Z(e^-(p_9),e^+(p_{10}))$	LO
533	$t(\to q(p_3)+\bar q(p_4)+b(p_5))+\bar t(\to e^-(p_7)+\bar\nu(p_8)+b\ (p_6))+Z(e^-(p_9),e^+(p_{10}))$	LO

540	$H(b(p_3)+\bar b(p_4))+t(p_5)+q(p_6)$	NLO
541	$H(b(p_3)+\bar b(p_4))+\bar t(p_5)+q(p_6)$	NLO
544	$H(b(p_3)+\bar b(p_4))+t(\nu(p_5)+e^+(p_6)+b(p_7))+q(p_9)$	NLO
547	$H(b(p_3)+\bar b(p_4))+\bar t(e^-(p_5)+\bar\nu(p_6)+b(p_7))+q(p_9)$	NLO
550	$H(\gamma(p_3)+\gamma(p_4))+t(p_5)+q(p_6)$	NLO
551	$H(\gamma(p_3)+\gamma(p_4))+\bar t(p_5)+q(p_6)$	NLO
554	$H(\gamma(p_3)+\gamma(p_4))+t(\nu(p_5)+e^+(p_6)+b(p_7))+q(p_9)$	NLO
557	$H(\gamma(p_3)+\gamma(p_4))+\bar t(e^-(p_5)+\bar\nu(p_6)+b(p_7))+q(p_9)$	NLO
560	$Z(e-(p_3)+e+(p_4))+t(p_5)+q(p_6)$	NLO
561	$Z(e-(p_3)+e+(p_4))+\bar t(p_5)+q(p_6)$	NLO
562	$Z(e-(p_3)+e+(p_4))+t(p_5)+q(p_6)+f(p_7)$	LO
563	$Z(e-(p_3)+e+(p_4))+\bar t(p_5)+q(p_6)+f(p_7)$	LO
564	$Z(e-(p_3)+e+(p_4))+t(\to\nu(p_5)+e^+(p_6)+b(p_7))+q(p_8)$	NLO
566	$Z(e-(p_3)+e+(p_4))+t(\to\nu(p_5)+e^+(p_6)+b(p_7))+q(p_8)+f(p_9)$	LO
567	$Z(e-(p_3)+e+(p_4))+\bar t(\to e^-(p_5)+\bar\nu(p_6)+\bar b(p_7))+q(p_8)$	NLO
569	$Z(e-(p_3)+e+(p_4))+\bar t(\to e^-(p_5)+\bar\nu(p_6)+\bar b(p_7))+q(p_8)+f(p_9)$	LO
601	$H(b(p_3)+\bar b(p_4))+H(\tau^-(p_5)+\tau^+(p_6))$	LO
602	$H(b(p_3)+\bar b(p_4))+H(\gamma(p_5)+\gamma(p_6))$	LO
640	$t(p_3)+\bar t(p_4)+H(p_5)$	LO
641	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to\bar\nu(p_7)+e^-(p_8)+\bar b(p_6))+H(b(p_9)+\bar b(p_{10}))$	LO
644	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to\bar q(p_7)+q(p_8)+\bar b(p_6))+H(b(p_9)+\bar b(p_{10}))$	LO
647	$t(\to q(p_3)+\bar q(p_4)+b(p_5))+\bar t(\to\bar\nu(p_7)+e^-(p_8)+\bar b(p_6))+H(b(p_9)+\bar b(p_{10}))$	LO
651	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to\bar\nu(p_7)+e^-(p_8)+\bar b(p_6))+H(\gamma(p_9)+\gamma(p_{10}))$	LO
654	$t(\to\nu(p_3)+e^+(p_4)+b(p_5))+\bar t(\to\bar q(p_7)+q(p_8)+\bar b(p_6))+H(\gamma(p_9)+\gamma(p_{10}))$	LO
657	$t(\to q(p_3)+\bar q(p_4)+b(p_5))+\bar t(\to\bar\nu(p_7)+e^-(p_8)+\bar b(p_6))+H(\gamma(p_9)+\gamma(p_{10}))$	LO
661	$t(\to\nu(p_3)e^+(p_4)b(p_5))+\bar t(\to\bar\nu(p_7)e^-(p_8)\bar b(p_6))+H(W^+(p_9,p_{10})W^-(p_{11},p_{12}))$	LO
664	$t(\to\nu(p_3)e^+(p_4)b(p_5))+\bar t(\to\bar q(p_7)q(p_8)\bar b(p_6))+H(W^+(p_9,p_{10})W^-(p_{11},p_{12}))$	LO
667	$t(\to q(p_3)\bar q(p_4)b(p_5))+\bar t\to(\bar\nu(p_7)e^-(p_8)\bar b(p_6))+H(W^+(p_9,p_{10})W^-(p_{11},p_{12}))$	LO

800	$V\to(\chi(p_3)+\bar\chi(p_4))+f(p_5)$ [Vector Mediator]	NLO
801	$A\to(\chi(p_3)+\bar\chi(p_4))+f(p_5)$ [Axial Vector Mediator]	NLO
802	$S\to(\chi(p_3)+\bar\chi(p_4))+f(p_5)$ [Scalar Mediator]	NLO
803	$PS\to(\chi(p_3)+\bar\chi(p_4))+f(p_5)$ [Pseudo Scalar Mediator]	NLO
804	$GG\to(\chi(p_3)+\bar\chi(p_4))+f(p_5)$ [Gluonic DM operator]	NLO
805	$S--(\chi(p_3)+\bar\chi(p_4))+f(p_5)$ [Scalar Mediator, mt loops]	NLO
820	$V\to(\chi(p_3)+\bar\chi(p_4))+\gamma(p_5)$ [Vector Mediator]	NLO + F
821	$A\to(\chi(p_3)+\bar\chi(p_4))+\gamma(p_5)$[Axial Vector Mediator]	NLO + F
822	$S\to(\chi(p_3)+\bar\chi(p_4))+\gamma(p_5)$[Scalar Mediator]	NLO + F
823	$PS\to(\chi(p_3)+\bar\chi(p_4))+\gamma(p_5)$[Pseudo Scalar Mediator]	NLO + F
840	$V\to(\chi(p_3)+\bar\chi(p_4))+f(p_5)+f(p_6)$ [Vector Mediator]	LO
841	$A\to(\chi(p_3)+\bar\chi(p_4))+f(p_5)+f(p_6)$ [Axial Vector Mediator]	LO
842	$S\to(\chi(p_3)+\bar\chi(p_4))+f(p_5)+f(p_6)$ [Scalar Mediator]	LO
843	$PS\to(\chi(p_3)+\bar\chi(p_4))+f(p_5)+f(p_6)$ [Pseudo Scalar Mediator]	LO
844	$GG\to(\chi(p_3)+\bar\chi(p_4))+f(p_5)+f(p_6)$ [Gluonic DM operator]	LO
845	$V\to(\chi(p_3)+\bar\chi(p_4))+\gamma(p_5)+f(p_6)$ [Vector Mediator]	LO
846	$A\to(\chi(p_3)+\bar\chi(p_4))+\gamma(p_5)+f(p_6)$ [Axial Vector Mediator]	LO
847	$S\to(\chi(p_3)+\bar\chi(p_4))+\gamma(p_5)+f(p_6)$ [Scalar Mediator]	LO
848	$PS\to(\chi(p_3)+\bar\chi(p_4))+\gamma(p_5)+f(p_6)$ [Pseudo Scalar Mediator]	LO
902	Check of Volume of 2 particle phase space	
903	Check of Volume of 3 particle phase space	
904	Check of Volume of 4 particle phase space	
905	Check of Volume of 5 particle phase space	
906	Check of Volume of 6 particle phase space	
908	Check of Volume of 8 particle phase space	
909	Check of Volume of 4 particle massive phase space	
910	Check of Volume of 3 particle (2 massive) phase space	
911	Check of Volume of 5 particle W+t (with decay) massive phase space	
912	Check of Volume of 5 particle W+t (no decay) massive phase space	
913	Check of Volume of 5 particle W+t+g (in decay) massive phase space	
914	Check of Volume of 5 particle W+t+g (in production) massive phase space	

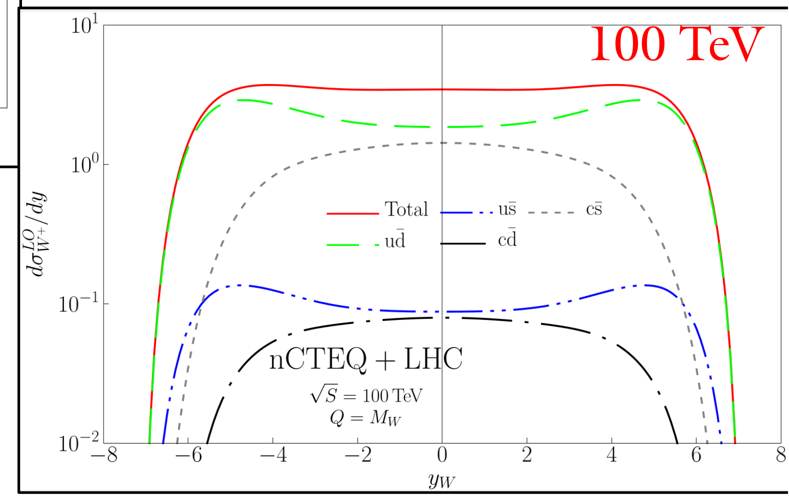
W+ Production at FCC



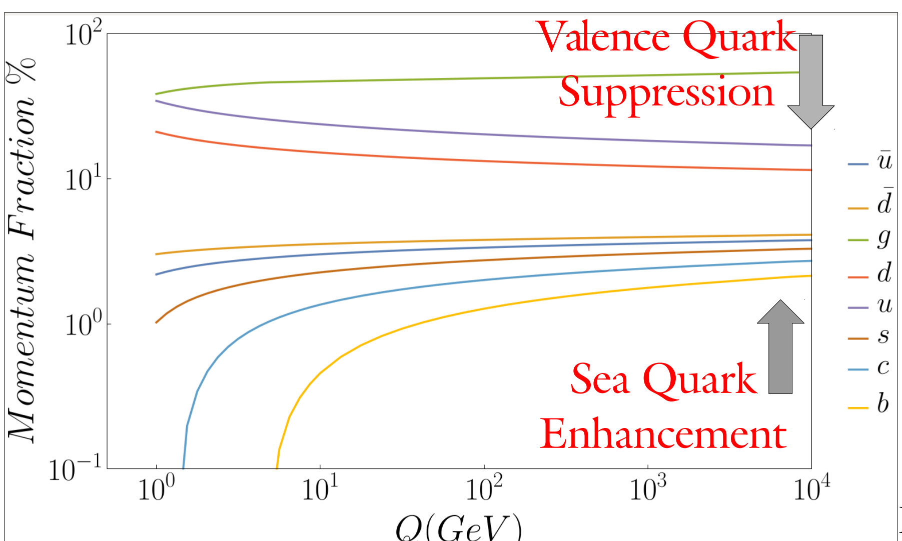


LHC
8TeV

FCC



- The Future Circular Collider
  - Proposed as a future hadron-hadron collider
  - Energies pushing 100 TeV



- Using nCTEQ+LHC to study W production
  - At FCC energies, $c\bar{s}$ is nearly as large as $u\bar{d}$
    - Especially at central rapidity
  - Fitting the strange quark PDF would have significant impact on this measurement

Special Thanks to:

Dissertation Committee

F. Olness (advisor)*

S. Sekula (chair)

K. Hornbostel

J. Owens*

J. Ye

nCTEQ Collaborators*

D.B. Clark

T. Jezo

C. Keppel

K. Kovarik

A. Kusina

F. Lyonnet

J. Morfin

I. Schienbein

J.Y. Yu

xFitter Collaboration

Conclusions

- Inclusion of W/Z LHC data improves nCTEQ15 fit
  - Better description of shape
    - Normalization important
      - This can be adjusted post-fit
  - Data coverage in new kinematic region eliminates extrapolation
    - Improves shape of fit in positive rapidity
  - Reweighting analysis did indicate direction of new fit
- nCTEQ parameterization remains overly restrictive
  - Fixed strange and anti-strange quark contributions
  - κ parameter from the proton fit
- Fitting normalization is necessary
  - Greatly reduces χ^2
  - Allows opening strange quark parameters

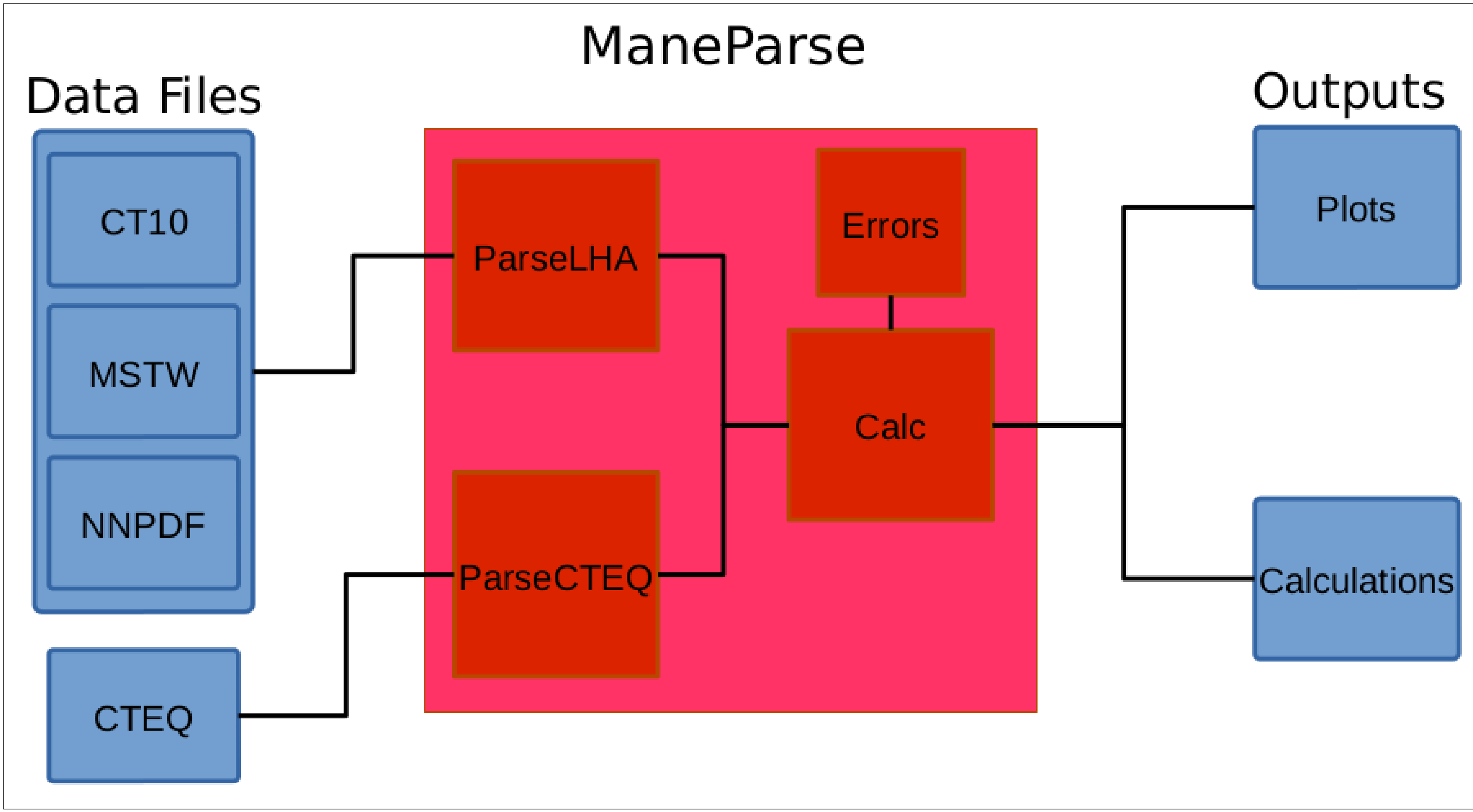
Additional Slides

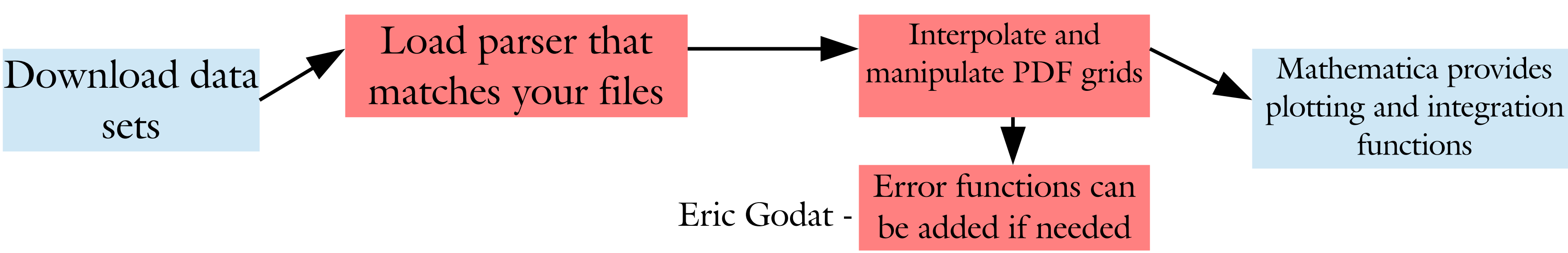
ManeParse

Development and Purpose

- Lightweight PDF Reader for multiple collaborations
  - LHAPDF6
  - CTEQ PDS
- Custom 4-point Lagrange Interpolation Routine
  - Fast, reliable, transparent
  - Adds continuity to otherwise discrete grids
- Mathematica provides user-friendly plotting and calculation functions
- Able to use multiple error techniques
  - Hessian
  - Monte Carlo
- Observables such as Luminosities and Cross Sections are calculable as well

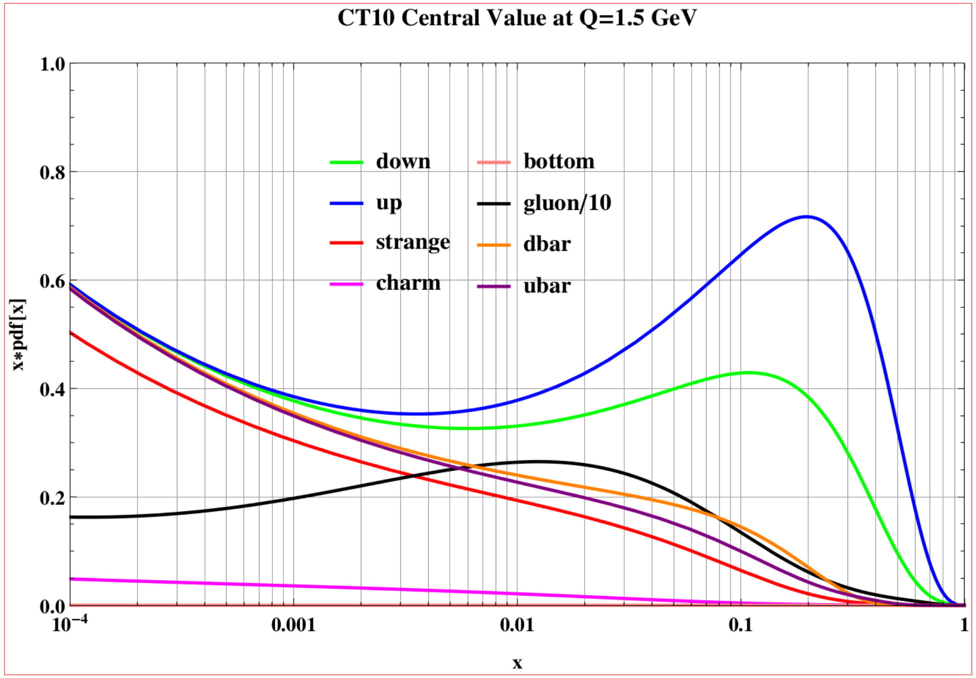
Schematic Overview

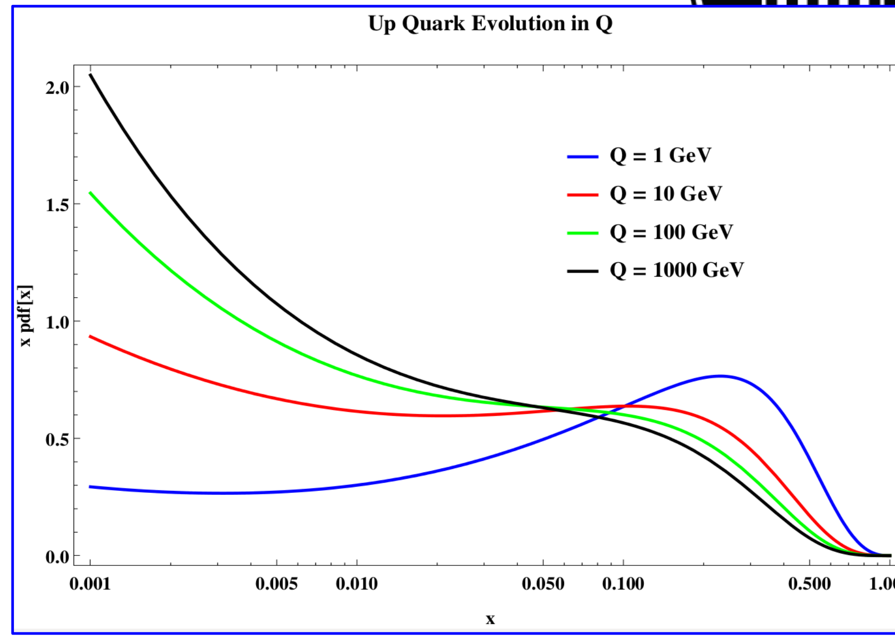




Feature Examples

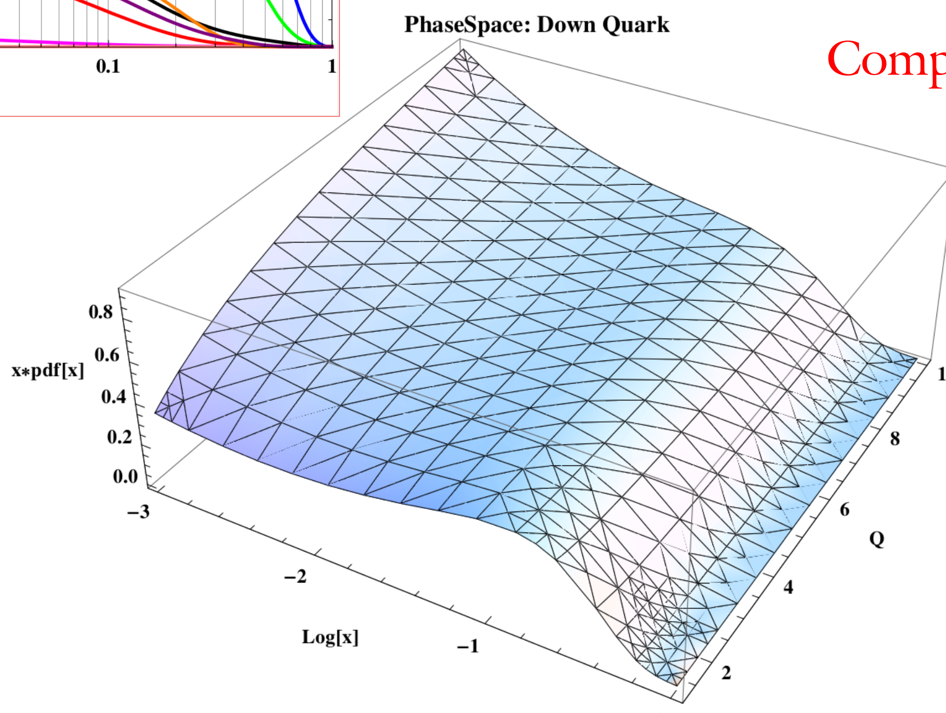
Plot of Multiple Partons for a Single PDF



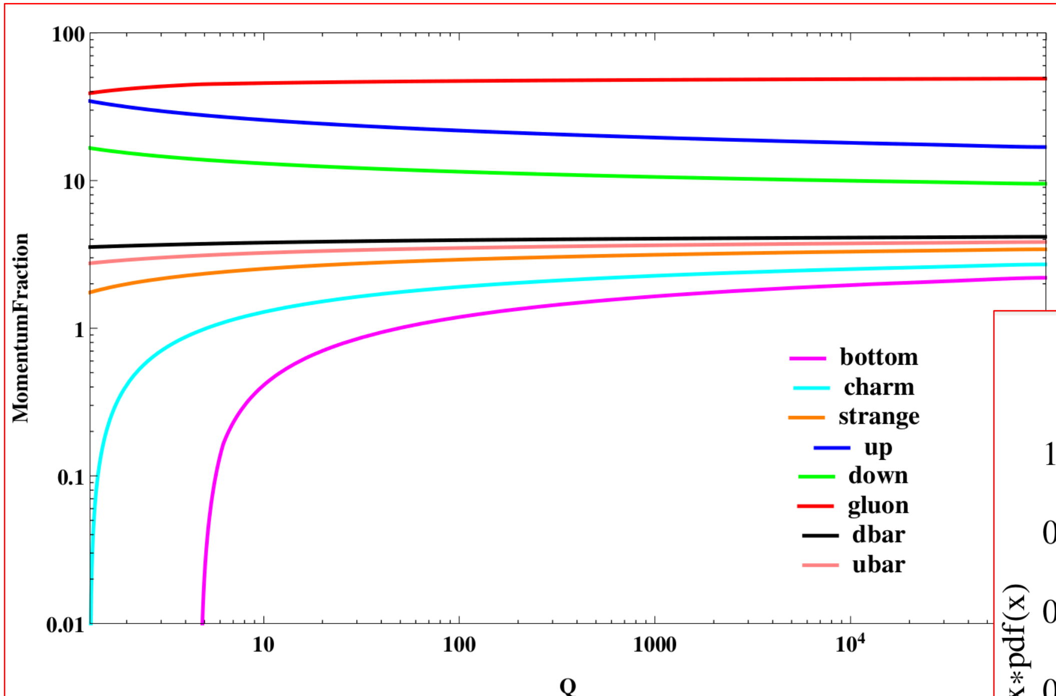


Compare PDFs at different energies

Visualize Phase Space with 3D plotting

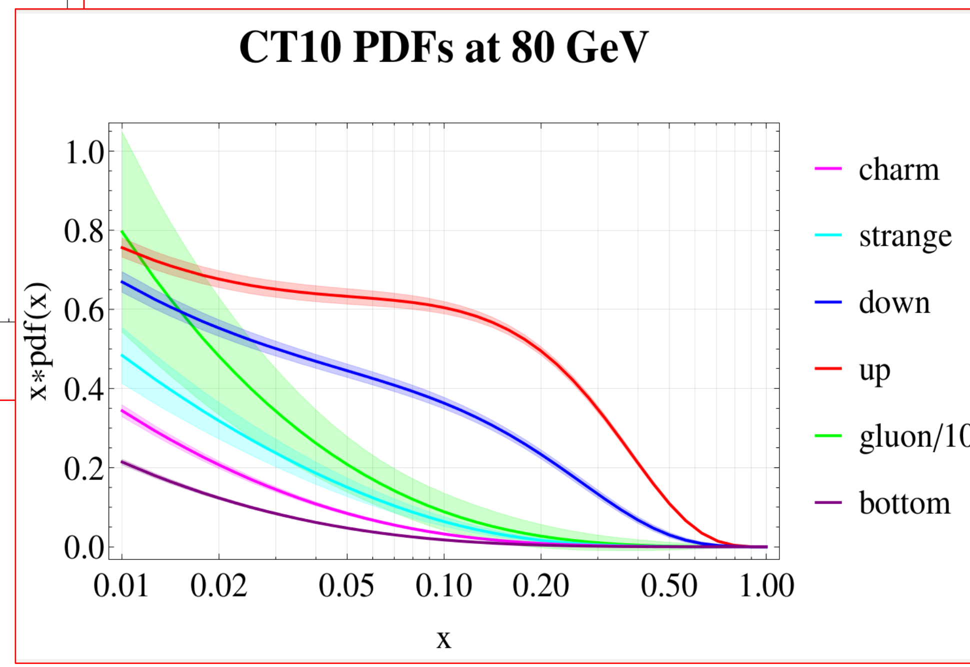


Feature Examples



Full sets of PDFs inside Mathematica

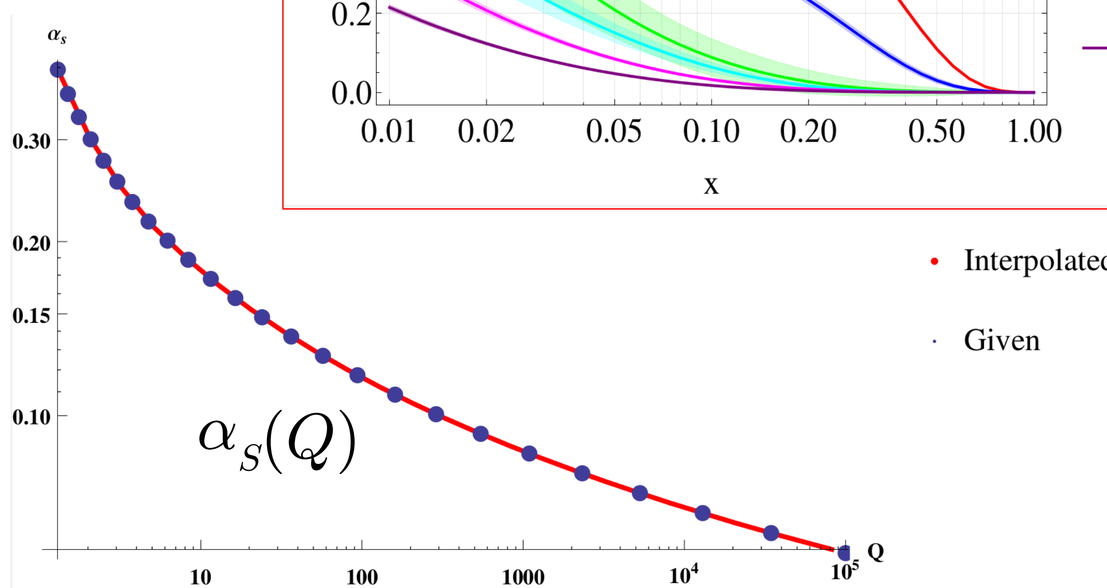
Easy to manipulate



Momentum Sum Rule provides a good check for interpolation errors

$$1=\sum_i \int_0^1 x \cdot pdf_i(x,Q)\,dx$$

Proper α_s is essential for NLO+ calculations



Contributions and Relationship

- ManeParse was developed exclusively by myself and Ben Clark under the supervision of Fred Olness
- My contributions:
  - LHAPDF6 Reader and Interface
    - The primary operating mode for most users
  - Designed majority of user functions
    - Wrote documentation for User Manual
  - Heavily contributed to content of the paper
- ManeParse was used extensively throughout this work
  - Cross checks
  - Visualization of results
  - FCC Prediction

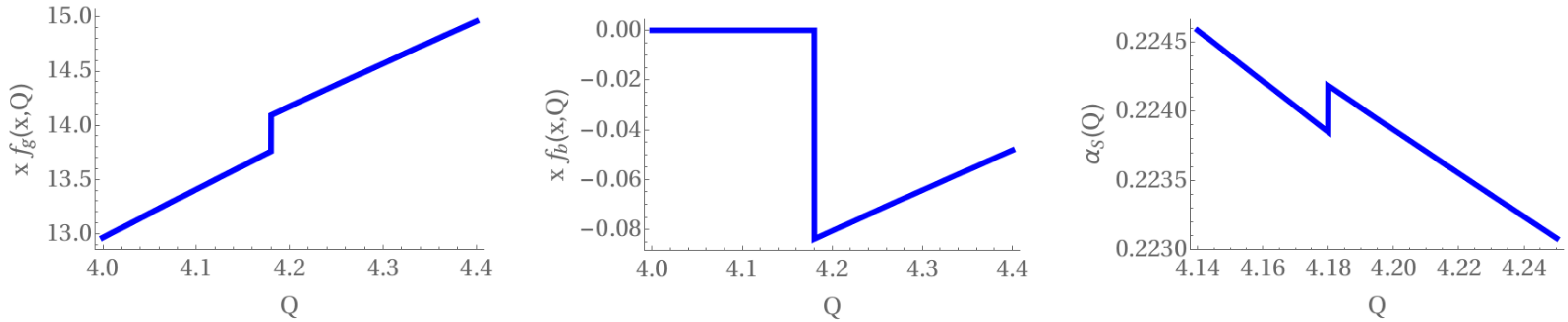
Heavy Flavor Variable Number Scheme

HFVNS

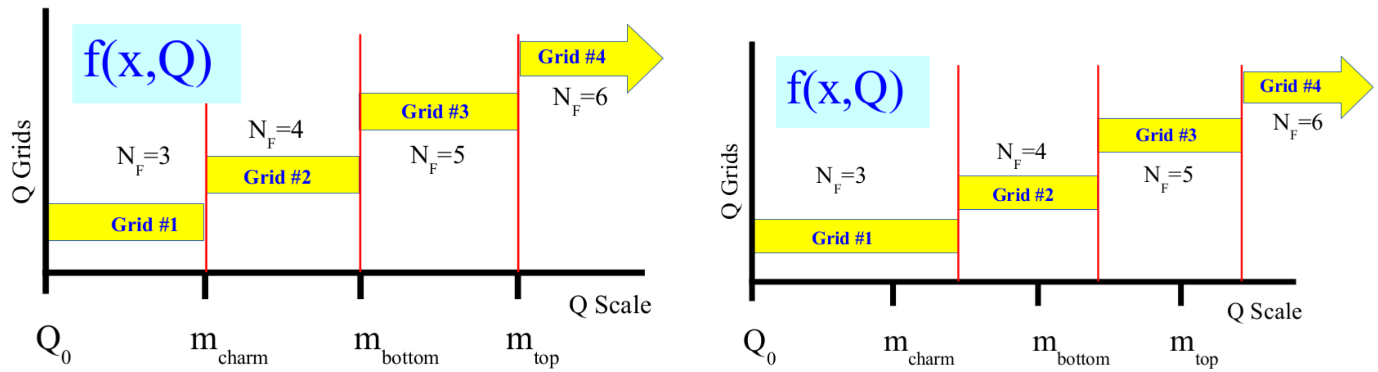
- Study the effect of shifting the mass thresholds for the heavy quarks when fitting PDFs to data
  - Done in xFitter in collaboration with xFitter development team
  - Charm and Bottom thresholds

HFVNS

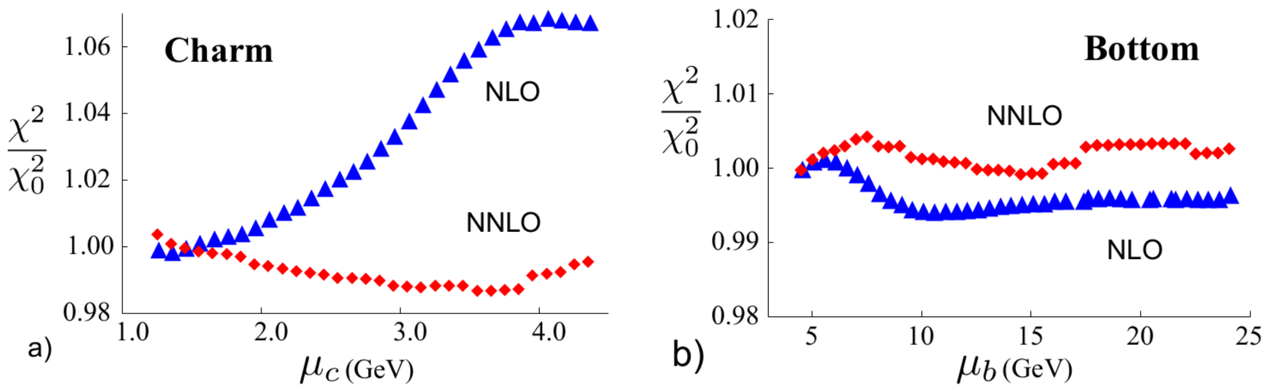
At NNLO, PDFs are discontinuous at mass thresholds for the quarks



This can be smoothed for fitting to data by shifting the mass threshold above the quark mass



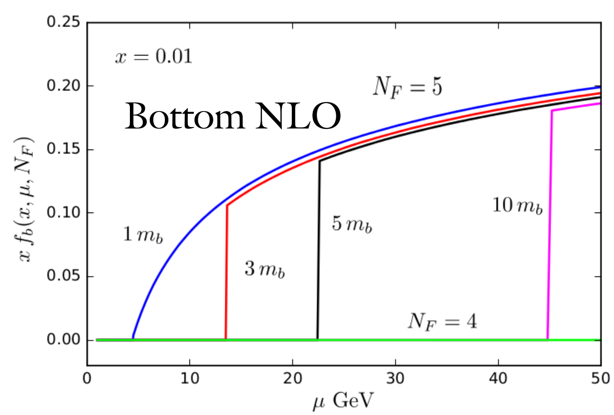
Fitting Heavy Quarks

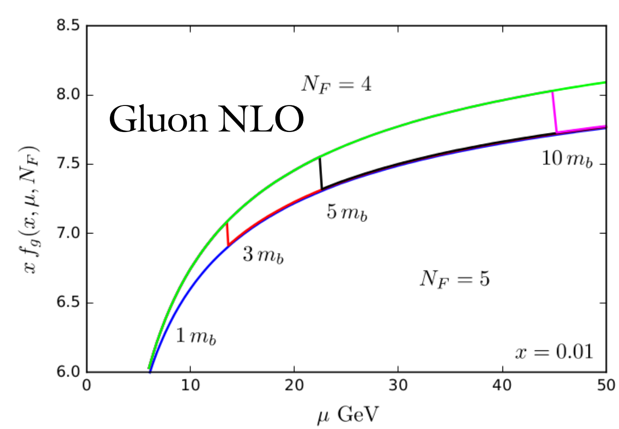


- Charm
  - NLO: Optimal threshold ~ m_c
    - Strong preference (~6%)
  - NNLO: No obvious threshold
    - Little variation (~1%)
- Bottom
  - NLO: Optimal threshold ~ 2mb
    - Little Preference (<1%)
  - NNLO: Optimal threshold ~ mb
    - Little Preference (<1%)

N_f dependent PDF

- Transition between number schemes
- Allows more flexibility in choice of number scheme





Contributions and Relationship

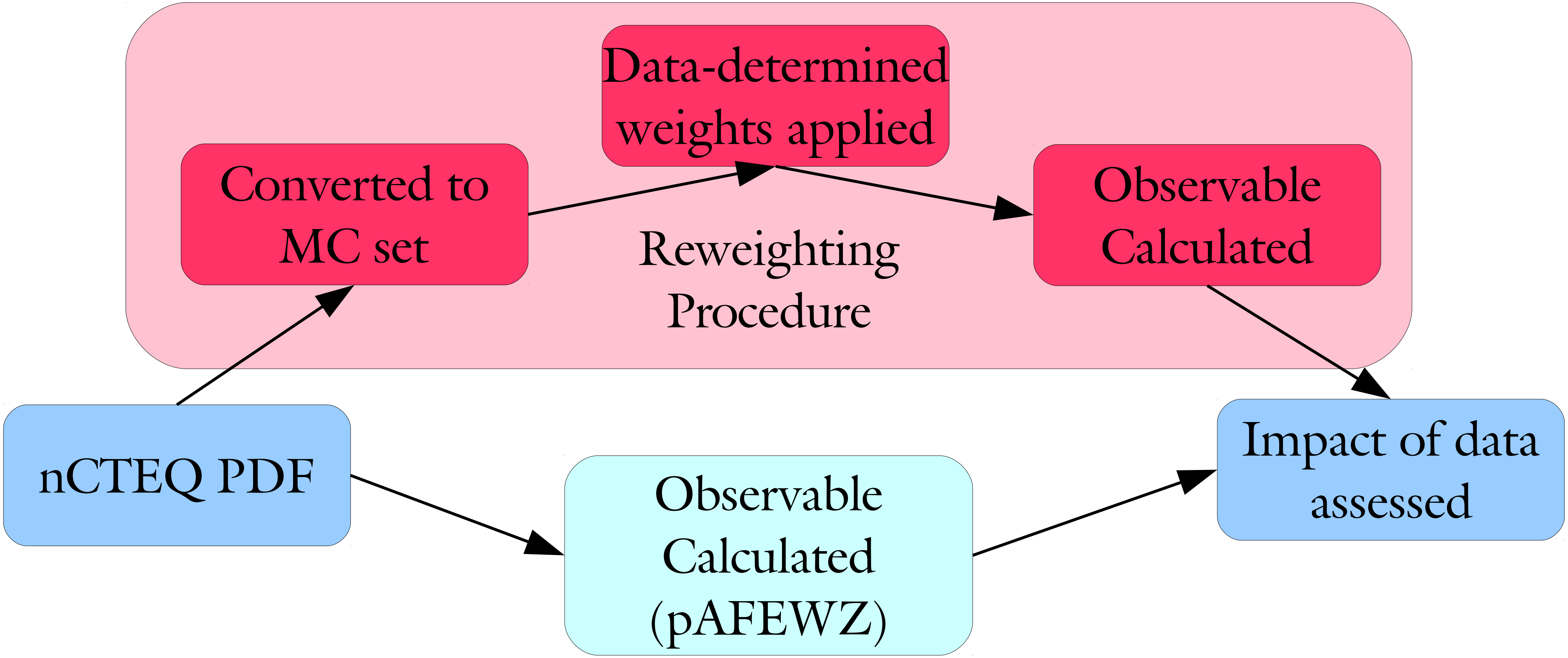
- Contributions:
  - Assisting with xFitter runs and analysis
  - Theory assessment for paper
  - Analysis of α_s for different flavor schemes in xFitter

    Mirrors the work done later for nCTEQ++ validation
- Led to a greater understanding of PDF fitting
  - nCTEQ fits at NLO not NNLO
    - Not applicable to nCTEQ++ and nCTEQ+LHC

Reweighting

Reweighting

- Determine the effect LHC pPb and PbPb W/Z data would have on existing nPDFs without a full refit



Reweighting Formalism

Bayesian Reweighting Technique

- Generate Replica: $f_k(x) = f_0 + \sum_i \frac{1}{2}(f_i^+ - f_i^-)R_{ik}$
- Calculate Giele-Keller weights: $w_k = \frac{e^{\frac{-1}{2}\chi_k^2}}{\frac{1}{N_{rep}}\sum_i e^{\frac{-1}{2}\chi_i^2}}$
- Estimate weighted average and standard deviation:

$$< O >= \frac{1}{N_{rep}} \sum_k w_k O(f_k)$$

$$\delta < O >= \sqrt{\frac{1}{N_{rep}} \sum_k w_k (O(f_k) - < O >)^2}$$

Contributions and Relationship

- Reweighting determined that LHC W/Z Data would be worth adding to nCTEQ15 fit
  - New kinematic region previously unconstrained by data
  - Potential to open strange parameter
- My contributions:
  - Validation of reweighting results in ManeParse
  - Numerous contributions from my work on nCTEQ+LHC (post-publication)

Backup Slides

Determination of α_s

nCTEQ++:

Runge-Kutta numerical solution

Number of Quark Flavors – 4, 5, 6

FORTRAN:

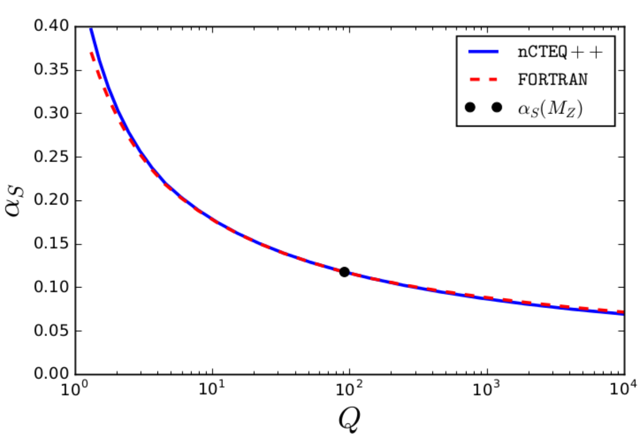
Truncated analytic series solution

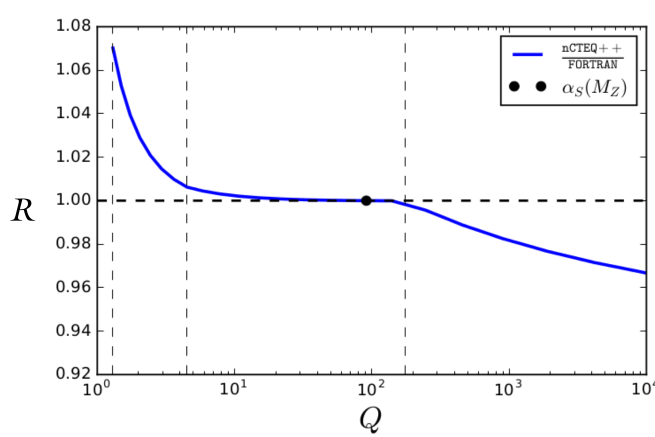
Number of Quark Flavors – 4, 5

Leading Order: $$\alpha_s(Q^2) = \frac{1}{b\ \ln(Q^2/\Lambda^2_{QCD})}$$

Higher Orders:

$$\frac{d\alpha_S}{d\ln(Q^2)} = \beta(\alpha_S(Q^2)) = -(b_0\alpha_S^2 + b_1\alpha_S^3 + b_2\alpha_S^4 + \ldots)$$



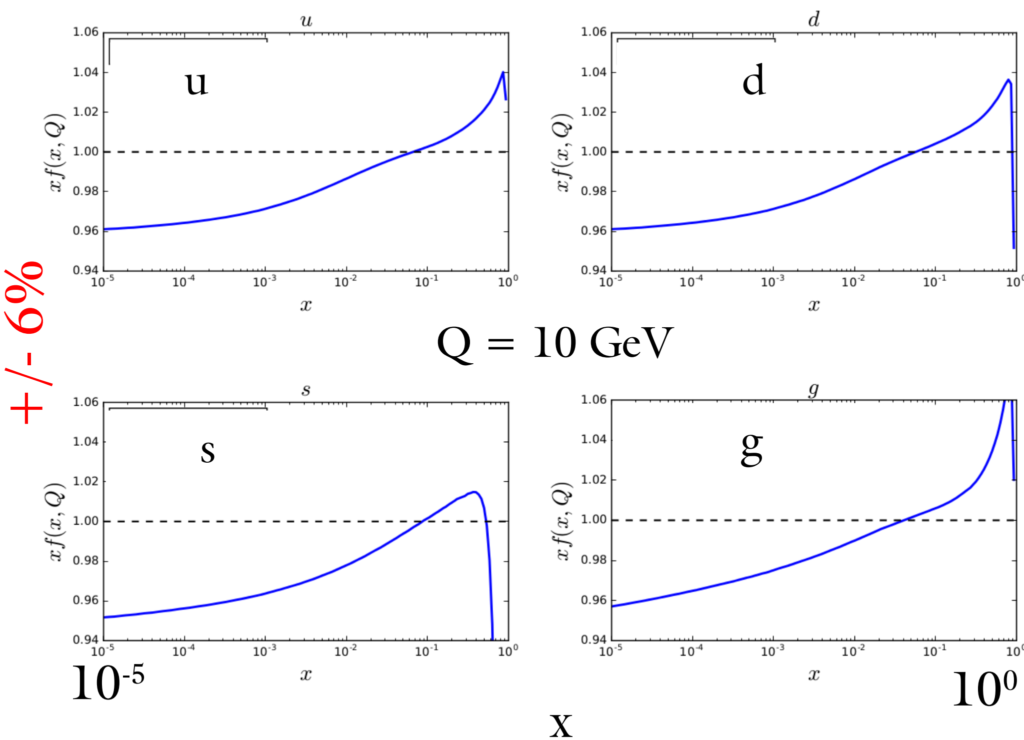


$$b_0 = \frac{33 - 2n_f}{12\pi}$$

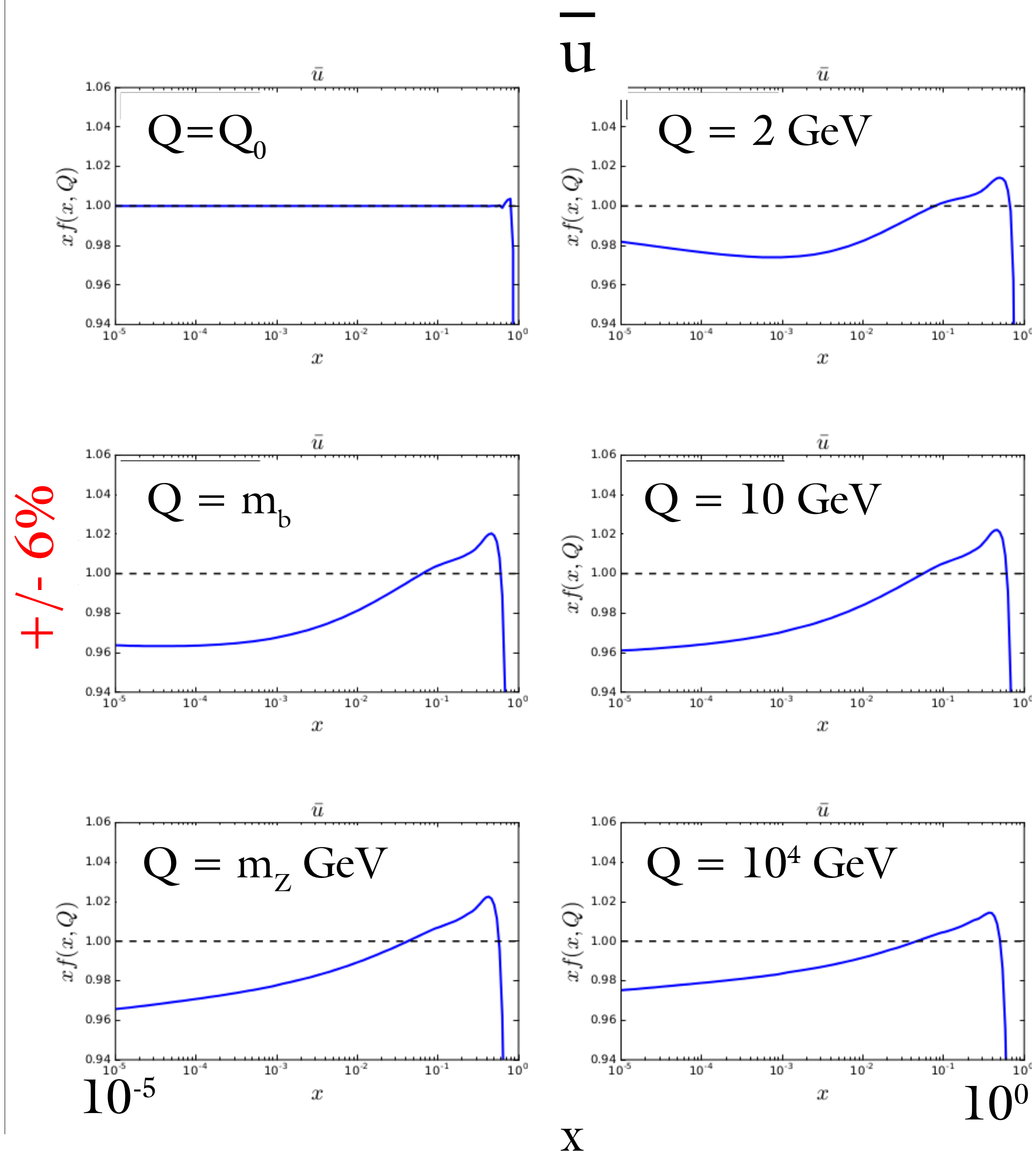
$$b_1 = \frac{153 - 19n_f}{24\pi^2}$$

Validation: Evolution

Ratio of PDF Evolution Codes

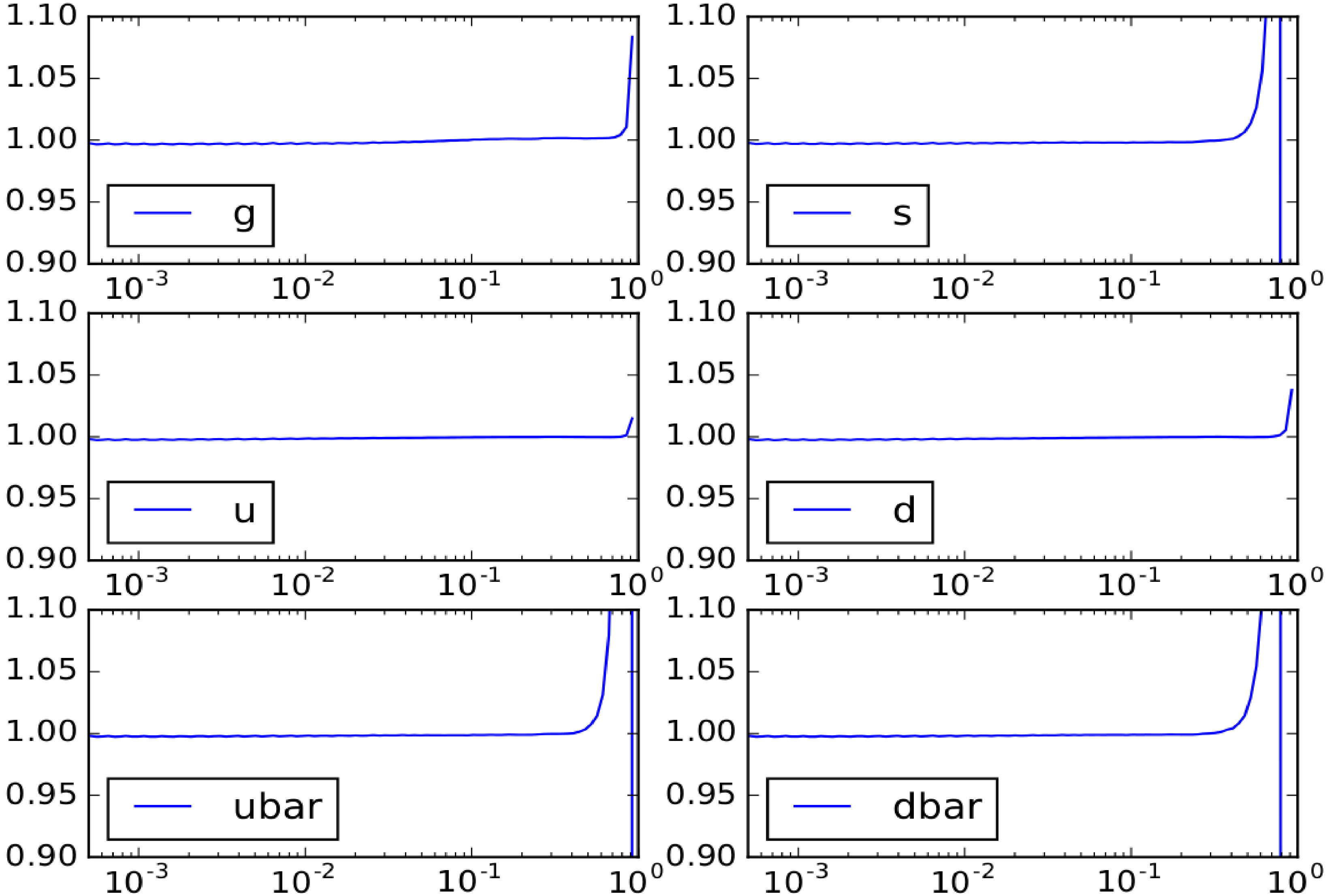


Ratio of PDF Evolution Codes



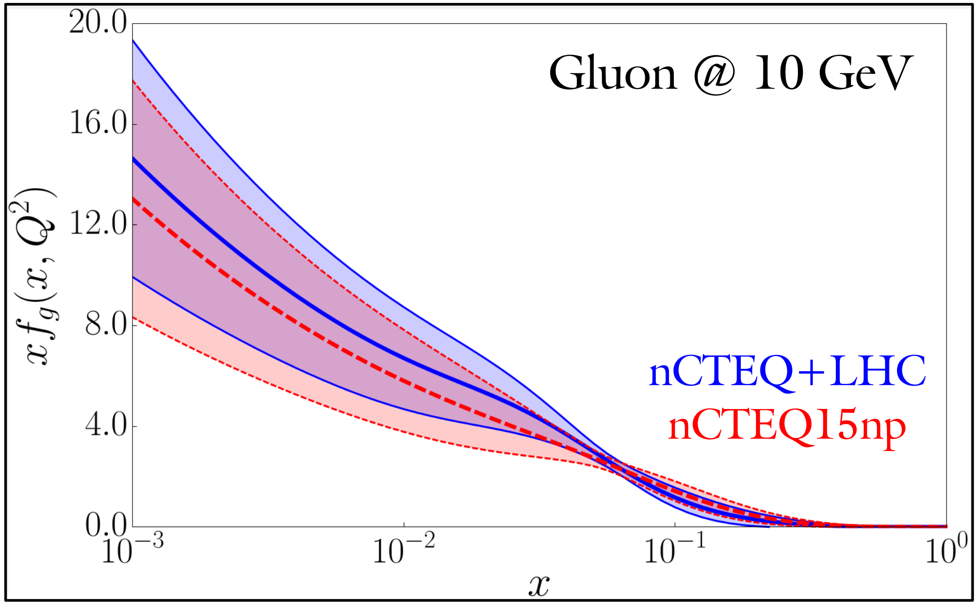
Validation: Evolution

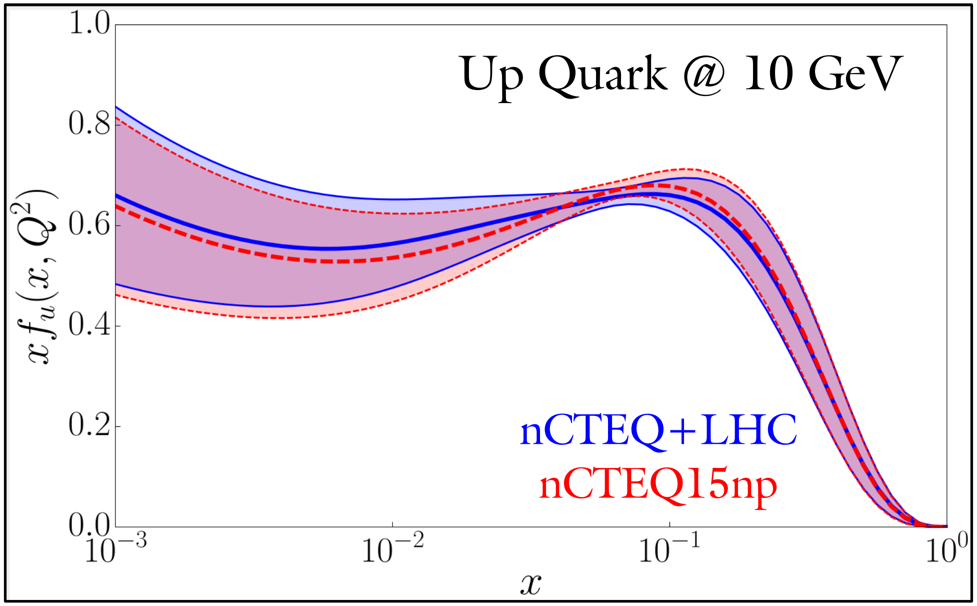
Ratio of PDF Evolution Codes
With the same α_s implemented



Error Band Estimate

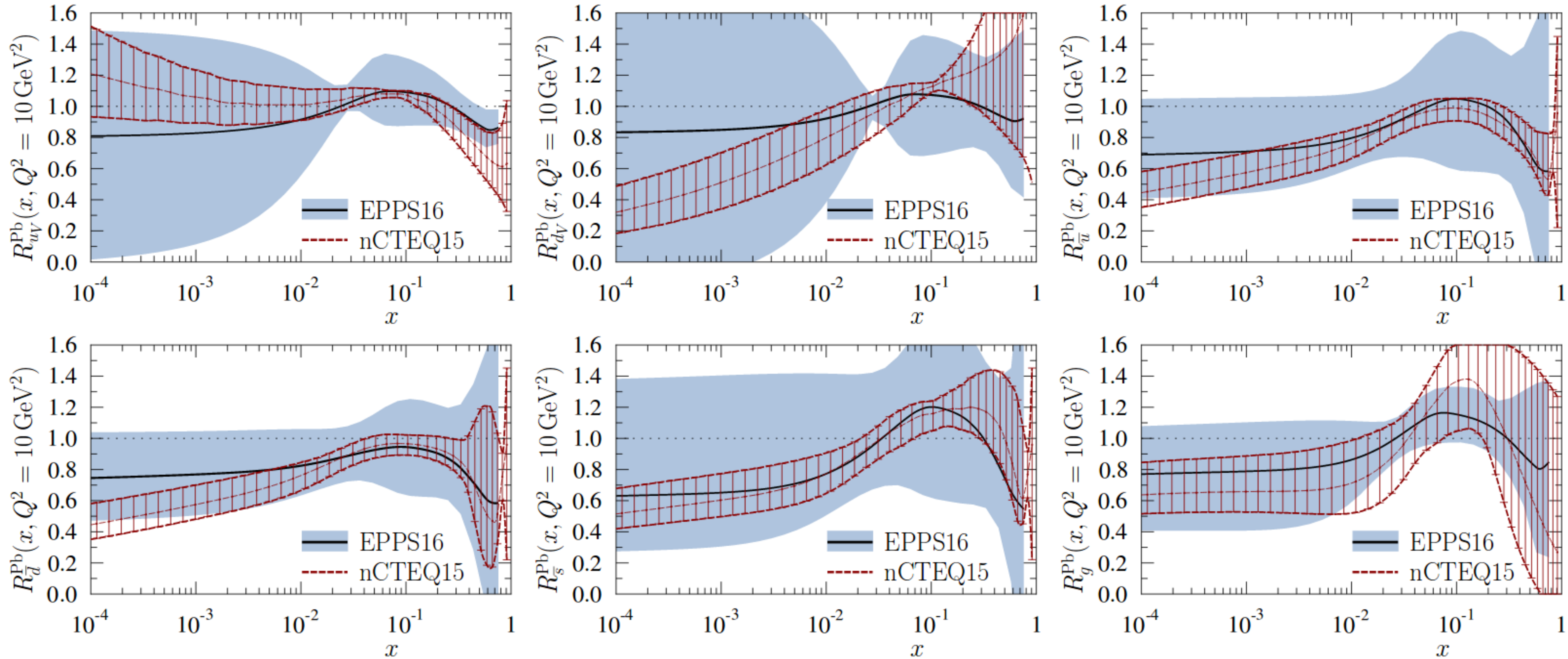
- nCTEQ++ currently lacks the capability to extract PDF error bands
  - Roughly estimate errors for nCTEQ+LHC by using nCTEQ15np error bands
    - Central value from nCTEQ+LHC +/- error from nCTEQ15np





EPPS16

- EPPS fits nuclear ratios, not nuclear PDFs
  - EPPS16 includes LHC data
    - CMS Di-jets
    - W/Z Production from CMS, Z Production from ATLAS
  - Also includes large number of CHORUS Pb Fixed Target DIS points (824)
- More than double the data points in nCTEQ15 (1789)



Source: arXiv 1612.05741

Nuclear Correction Regions

(1) Shadowing

- Destructive interference between virtual boson and nucleons

(2) Anti-Shadowing

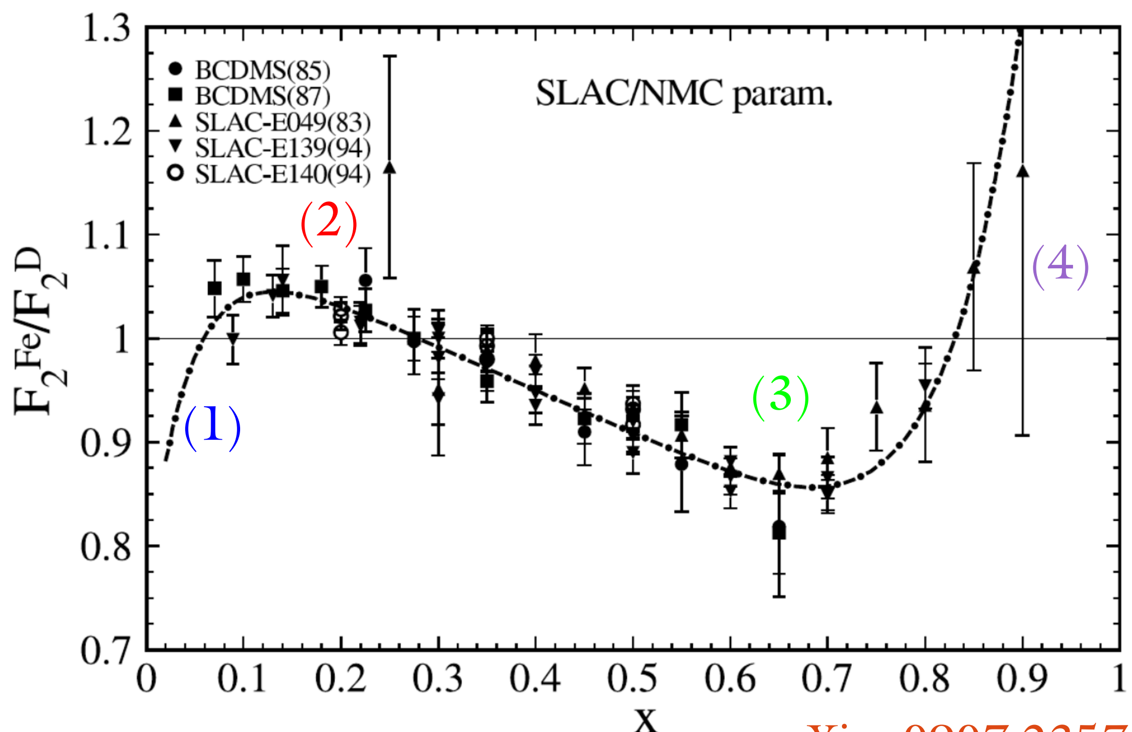
- Constructive interference between virtual boson and nucleons

(3) EMC Effect

- Discovered by European Muon Collaboration in 1983
- No definitive explanation

(4) Fermi Motion

- Quantum motion of nucleons

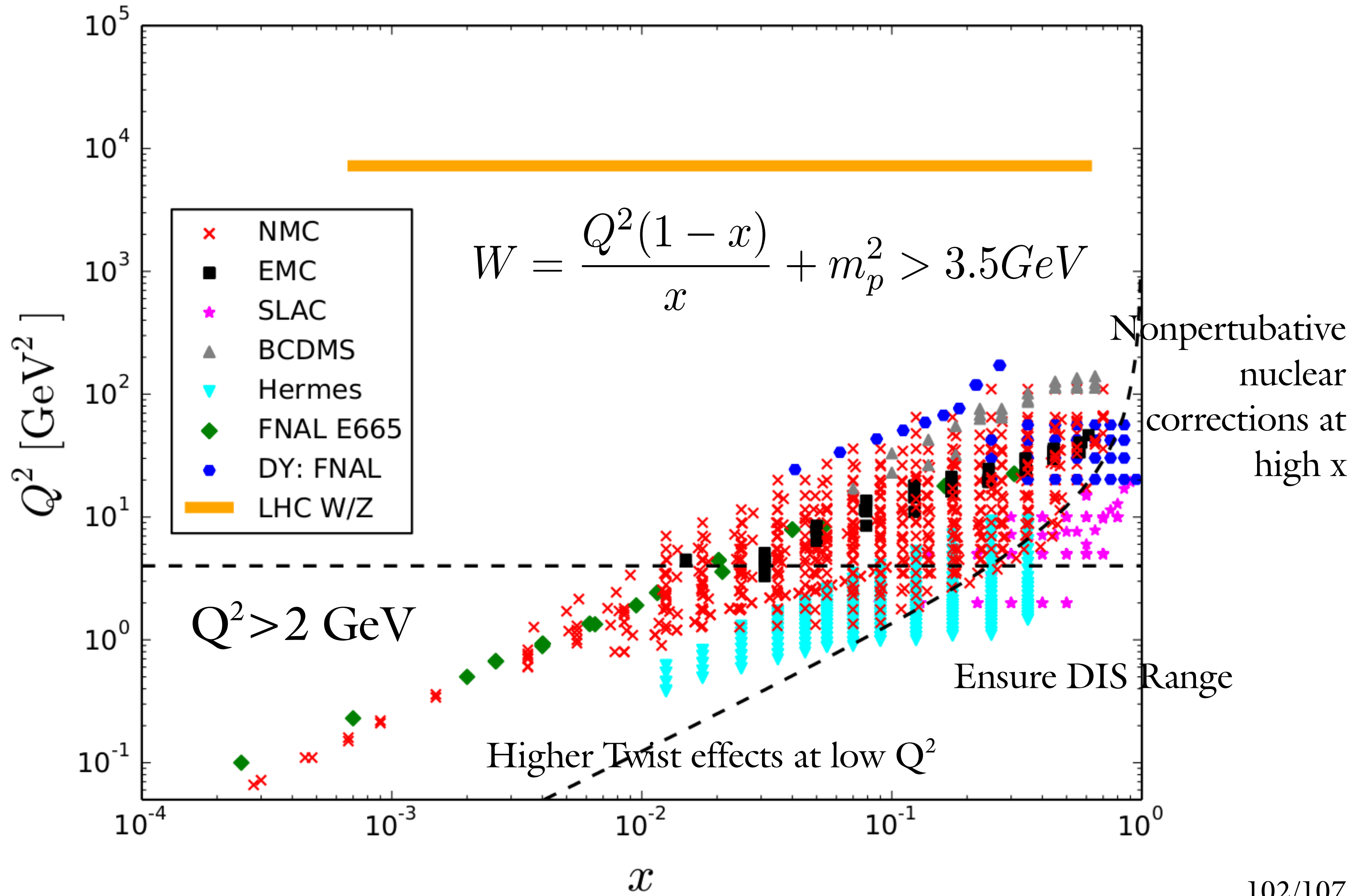


arXiv: 0907.2357

DIS Structure Functions

- The experimental observable related to hadronic structure
  - In the parton model, they can be mapped directly to PDFs
- DIS Cross Section:
  - $\frac{d^2\sigma^i}{dxdy} = (\frac{4\pi\alpha^2}{xyQ^2}\eta^i)\left\{y^2xF_1^i + (1 - y - \frac{x^2y^2M^2}{Q^2})\, F_2^i \mp (y - \frac{y^2}{2})\, xF_3^i\right\}$
  - $F_L = 0 \Rightarrow F_2 = 2xF_1$
    - This is known as the Callan-Gross relation
- Example:
  - $F_2^{ep} = \frac{4}{9}x[u + \bar{u} + c + \bar{c}] + \frac{1}{9}x[d + \bar{d} + s + \bar{s}]$

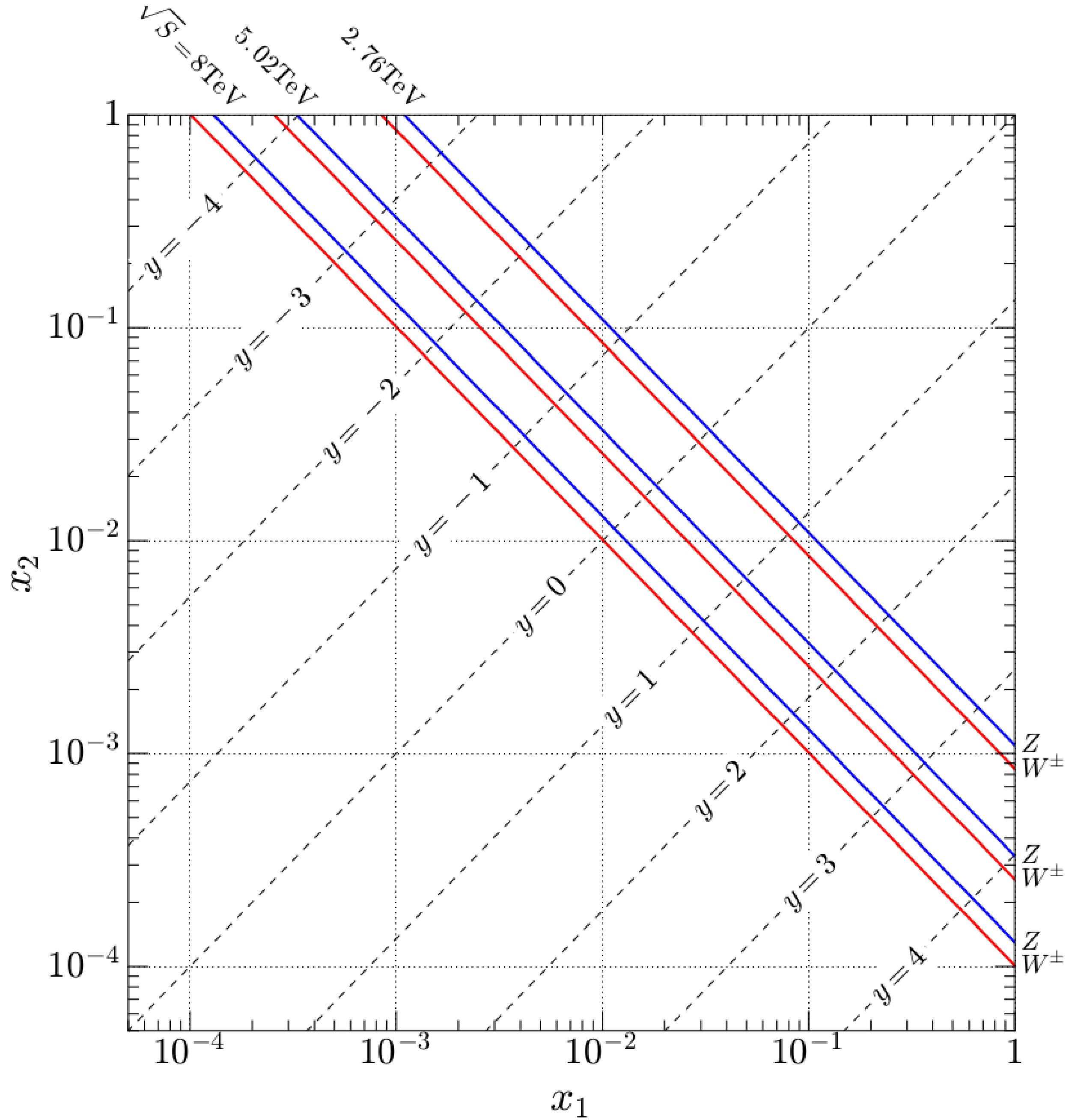
LHC Data vs Current Data Cuts



Twist

- Formally a quantum number describing the exponent on the mass term of the matrix element in the definition of the structure function
  - Determined solely through dimensional analysis
  - Related to the dimension and spin
  - Twist-2 is Leading Order
    - Twist-3 and Twist-4 are considered higher order
- Practically it describes the order (in $1/Q^2$) at which an effect is seen in an experiment
  - Gluon interactions within the nucleon are higher twist effects
  - High twist effects are suppressed by $1/Q^2$
  - Cuts at low Q^2 are designed to limit theses contributions
- Intuitively it describes how uncertain we are that a particular parton actually has the momentum that the PDF describes during the short range scattering process

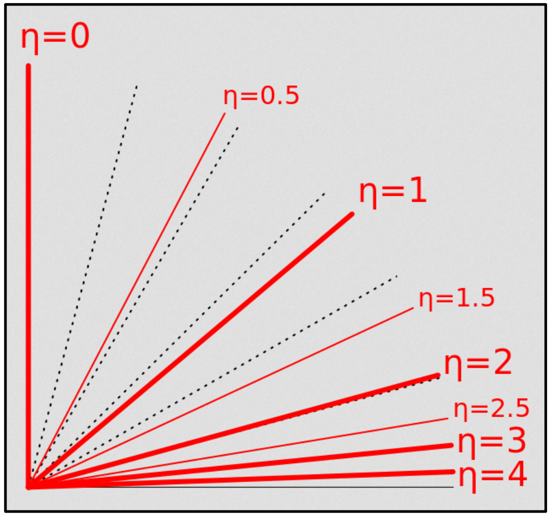
Relating x_1 and x_2 to Rapidity



x_2 is the Lead PDF

Collider Definitions

- Rapidity: $y = \frac{1}{2} ln \frac{E + p_z}{E - p_z}$
  - A measure of how far forward a particle is boosted
  - Pseudorapidity: $\eta = \frac{1}{2} ln \frac{\vec{p} + p_z}{\vec{p} - p_z}$
    - The massless equivalent of rapidity

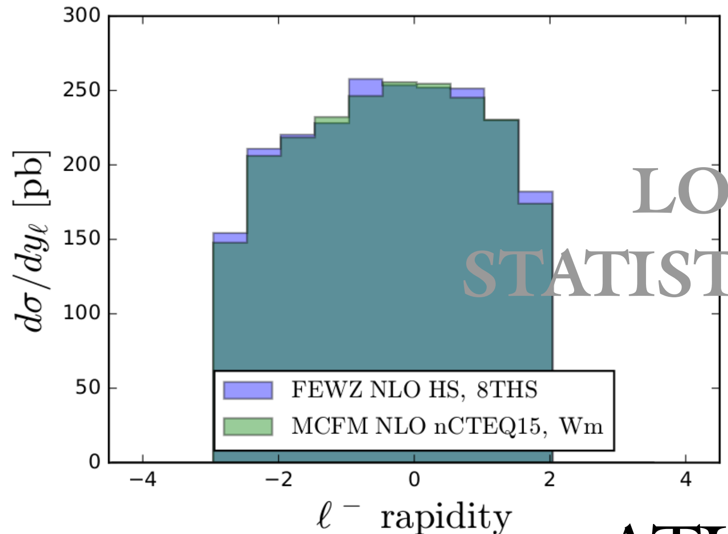


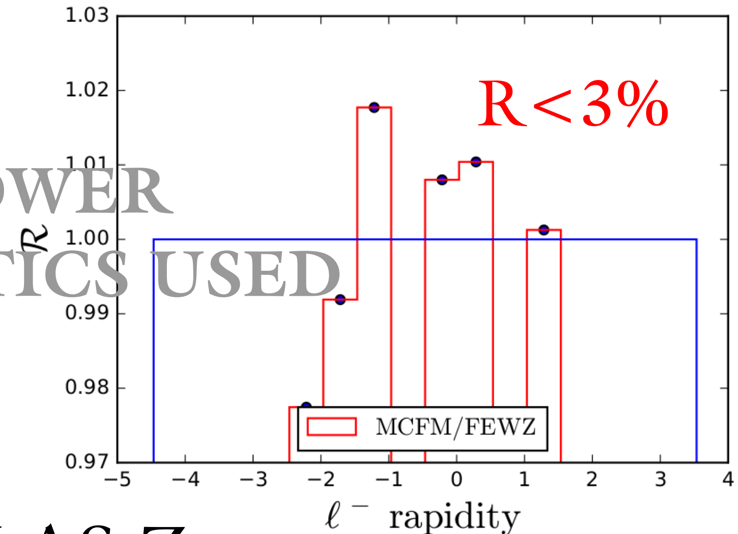
- Cross Section: $\sigma_{pp \to H+X} = \sum_{ij} \int_\tau^1 d\tau \frac{d\mathcal{L}_{ij}}{d\tau} \hat{\sigma}_{ij}(\hat{s})$
  - A measure of how likely an interaction will occur
- Luminosity: $\frac{d\mathcal{L}_{ij}}{d\tau}(\tau, \mu) = \frac{1}{1+\delta_{ij}} \int_\tau^1 \frac{1}{x} [f_i(x, \mu) f_j(\tau/x, \mu) + f_j(x, \mu) f_i(\tau/x, \mu)] dx$
  - Ratio of the number of events to the cross section
    - More luminosity = More Data

Image Source

Compare FEWZ-pp to MCFM-pp

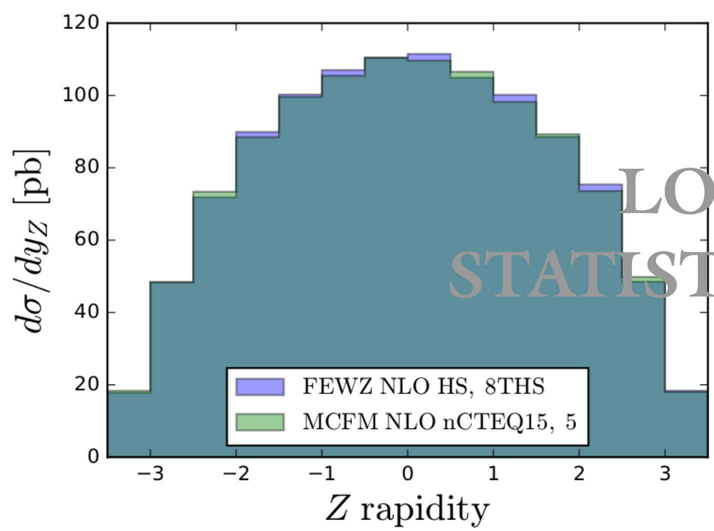
ATLAS W-

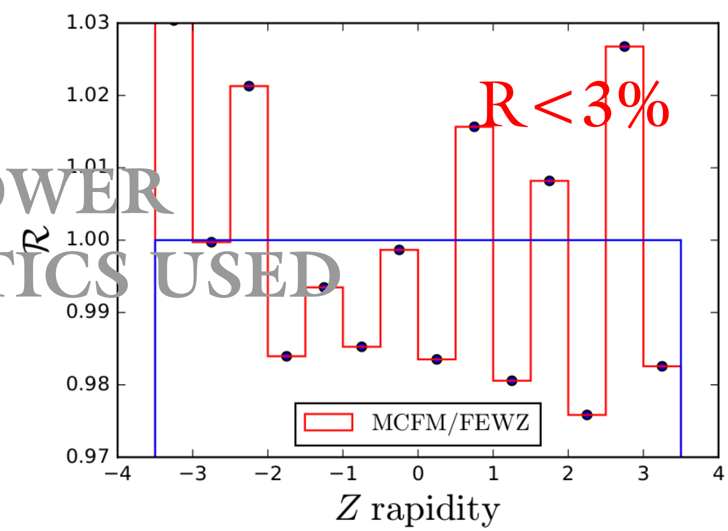




LOWER STATISTICS USED

ATLAS Z





LOWER STATISTICS USED

APPLgrid Technique

APPLGRID method

Eur.Phys.J.C66:503-524,2010.

- Step 1 (long run): Collect perturbative weights to grids .
  - binning (x_1, x_2, Q^2)
  - interpolation
  - initial flavours decomposition : $13 \times 13 \rightarrow \mathcal{L}$ $(\mathcal{L} \sim 10)$

$$\frac{d\hat{\sigma}^{ij}_{(p)}}{dX}(x_1, x_2, Q_F^2, Q_R^2;\ S) \xrightarrow{3D-grid} w^{(p)(l)}(x_1^m, x_2^n, Q^{2^k})\ (Q_R^2 \equiv Q_F^2)$$

- Step 2 ($\sim$ 10–100 ms): Convolute grid with PDF's .
  - integral $\rightarrow$ sum
  - any coupling, PDF

$$\frac{d\sigma}{dX} = \sum_p \sum_{l=0}^{L} \sum_{m,n,k} w^{(p)(l)}_{m,n,k} \left(\frac{\alpha_s(Q_k^2)}{2\pi}\right)^{p_l} F^{(l)}\left(x_{1m}, x_{2n}, Q_k^2\right)$$

Stolen from:
APPLGrid Talk at HERAFitter 2016